Learning Earth Systems Dynamics for Applications in Sustainability

AI Spring School 2026

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Associate Professor, Electrical Engineering Department, LUMS

SBASSE LUMS. April 4th, 2026

LUMS | Centre for Water Informatics and Technology

NCRA National Centre of Robotics & Automation | Agricultural Robotics Lab

xWIT

SMART WATER SOLUTIONS

Centre for Water Informatics & Technology

GMRC, WAPDA

GB EPA

IIASA

ICIMOD

PBL Netherlands Environmental Assessment Agency

Foreign & Commonwealth Office

ICTP

ADB

DAAD

PIK

Climate Analytics

Agriculture 4.0

Water Policy & Governance

Human-Water Systems

PARC

IRRIGATION Department Punjab

On Farm Water Management
Directorate General Agriculture

SAWiE

Ingredion

ALMOIZ INDUSTRIES LIMITED

FARMDĀR
Fruitful data for agriculture

PEPSICO

Nestlé

Aga Khan Agency for Habitat

BAYER

isif asia

NAMAL COLLEGE

Outline

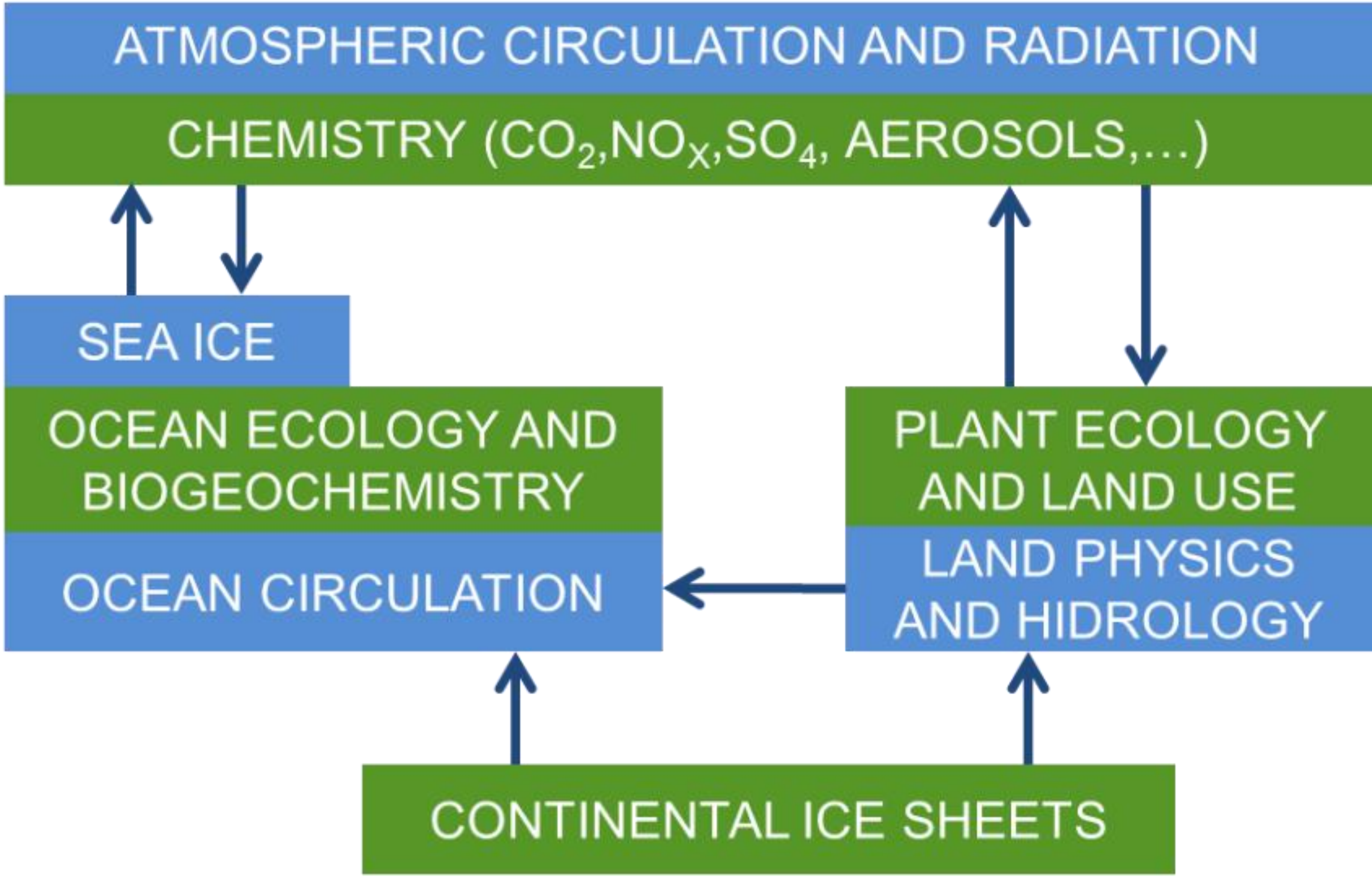
- Introduction & Motivation
- Earth Systems Models Complexity
- Discovering Unknown Dynamics
- Learning Nonlinearity
- Reducing High-Dimensionality
- Glimpse of Koopman Operator Theory
- Applications
  - Irrigation Scheduling (Field Scale, Canal-Command Scale, Basin Scale)
  - Cryosphere Monitoring (Seasonal Snowfall, Glacier Surging)
  - ~~Streamflow Prediction (Flood Early Warning, Seasonal release planning)~~
  - ~~Groundwater Management (Sustainability)~~

Joint Work with:

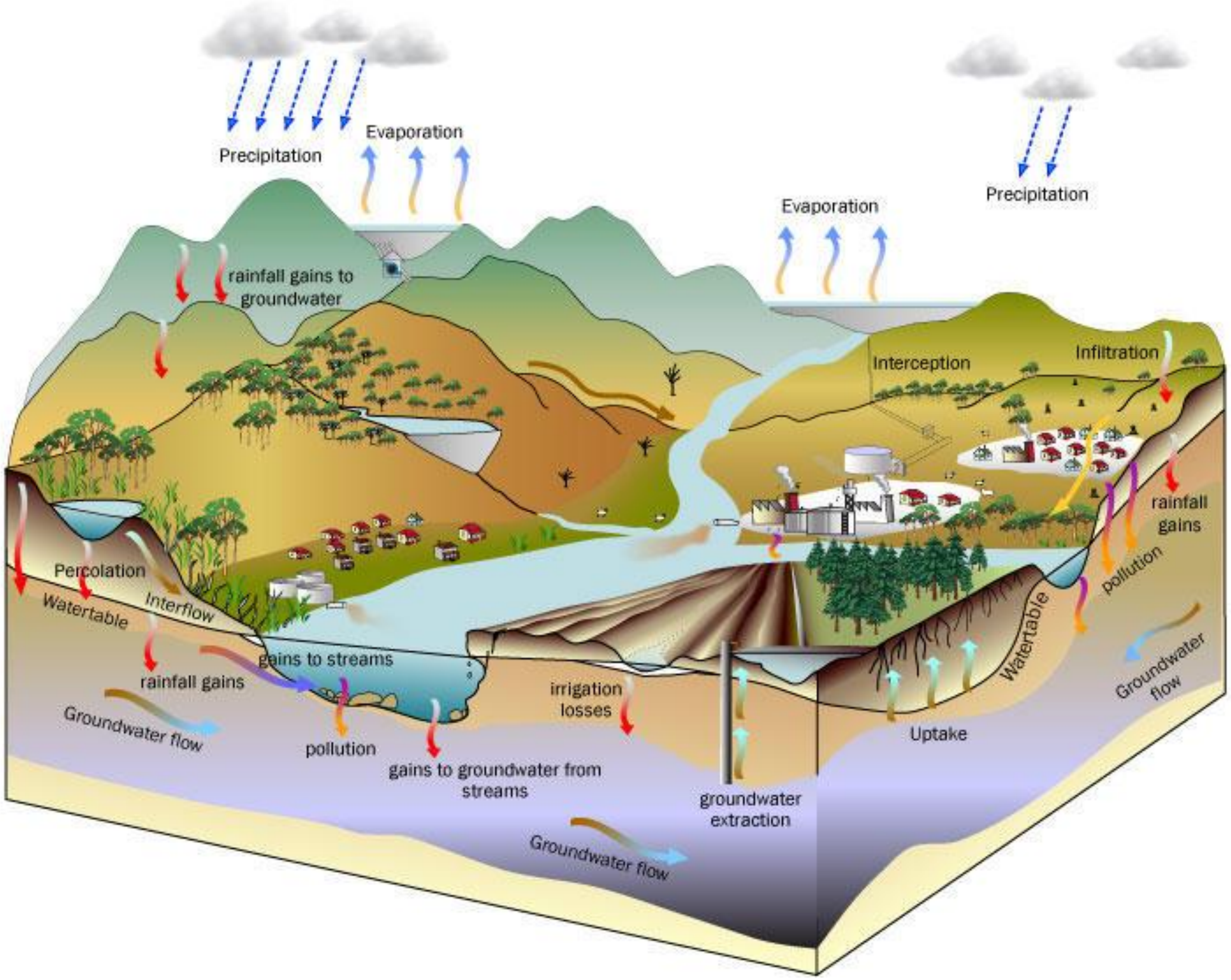
- PhD students
  - Hamza Rafique
  - Hassaan Ahmad
  - Ahmad Haseeb Rabbani
- Research Associates
  - Haseeb Ahmad
- Undergraduate Students
  - Haris Rathore
  - Ali Anser Jaffri

What I will not cover today:

- Agricultural robotics (UAVs, UGVs, USVs) (with Dr. H. Jaleel, Dr. M. Taj)
- Integrated Assessment Modeling / Climate Change Governance (with Dr. M. Awais)
- Socio-hydrological Systems Analysis (with Dr. Talha Manzoor)
- Hydro-metereological sensor networks / Edge AI (with Z. Ahmad, Farhan Ammar)
- Flood Early Warning, Farmer Advisory services (with Ali A. Abbas, xWIT)

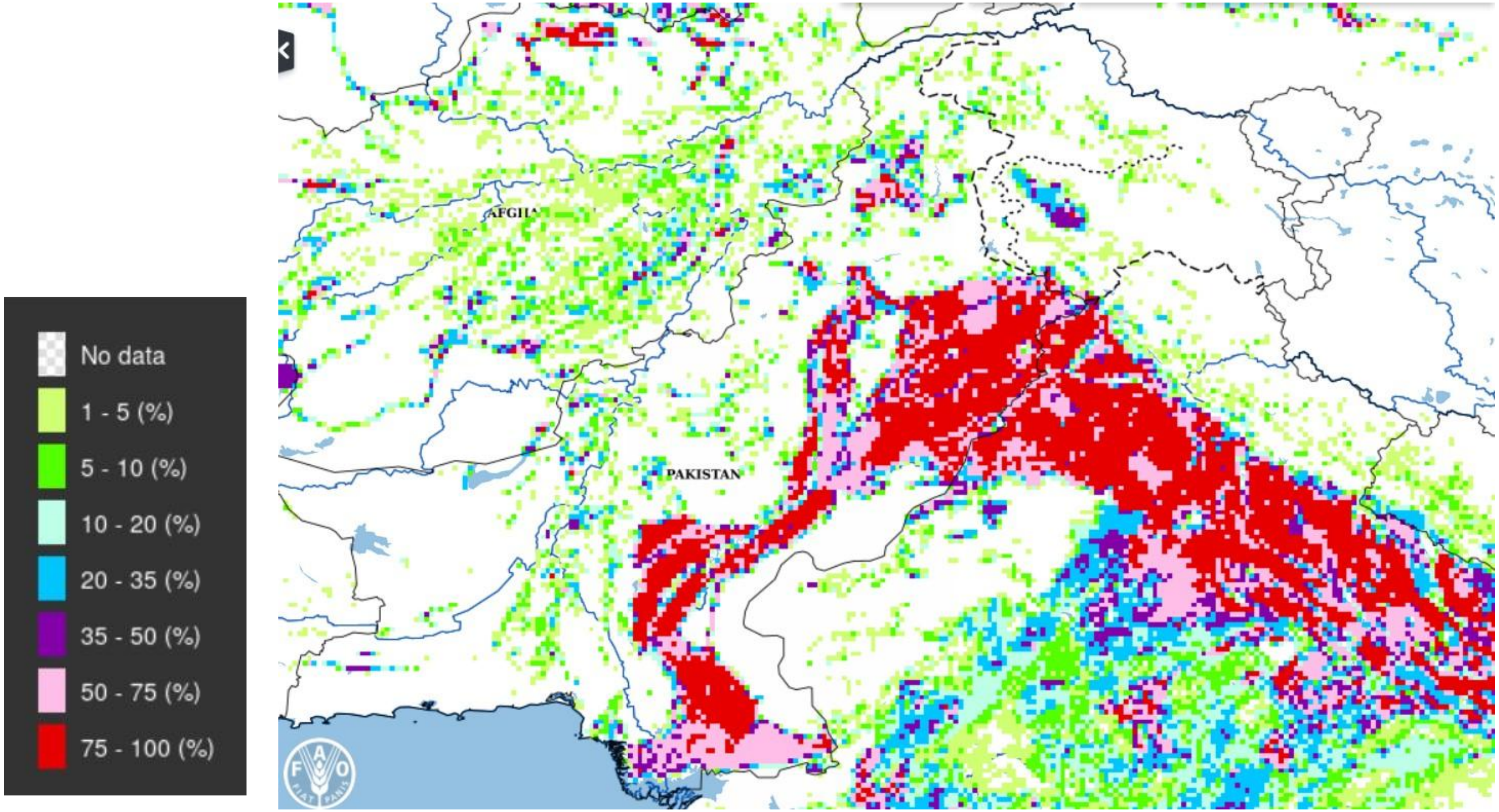


https://www.climateurope.eu/earth-system-modeling-a-definition/



https://www.geo.fu-berlin.de/en/v/iwrm/Implementation/water_and_the_physical_environment/water_cycle/index.html

Irrigation Intensity in the Indus Basin



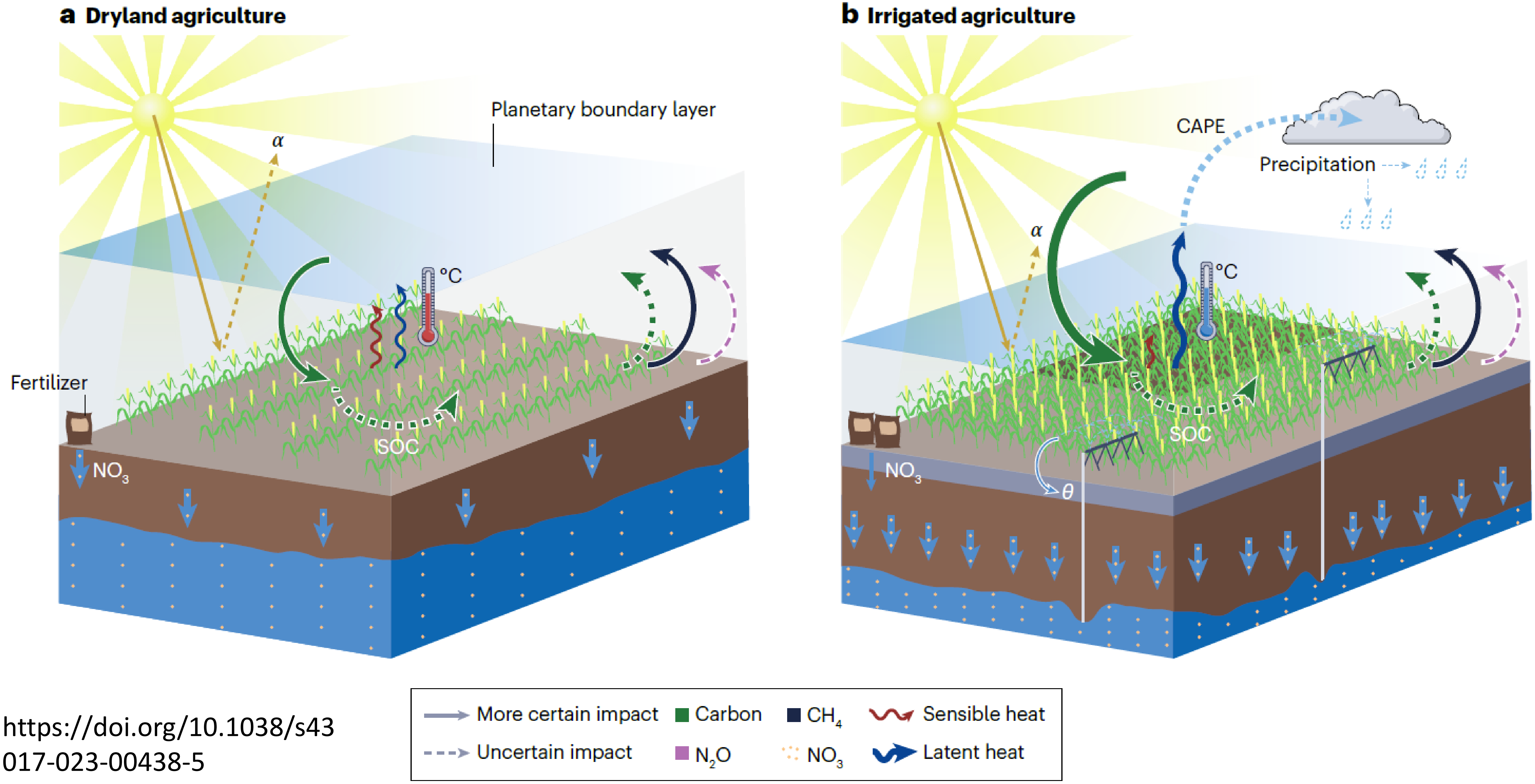
Indus Basin supports **a high proportion** of irrigated areas

Global Map of Irrigation Areas (Version 5, 2013)
Source: FAO.2024 AQUASTAT Core Database (accessed 17 May 2025)

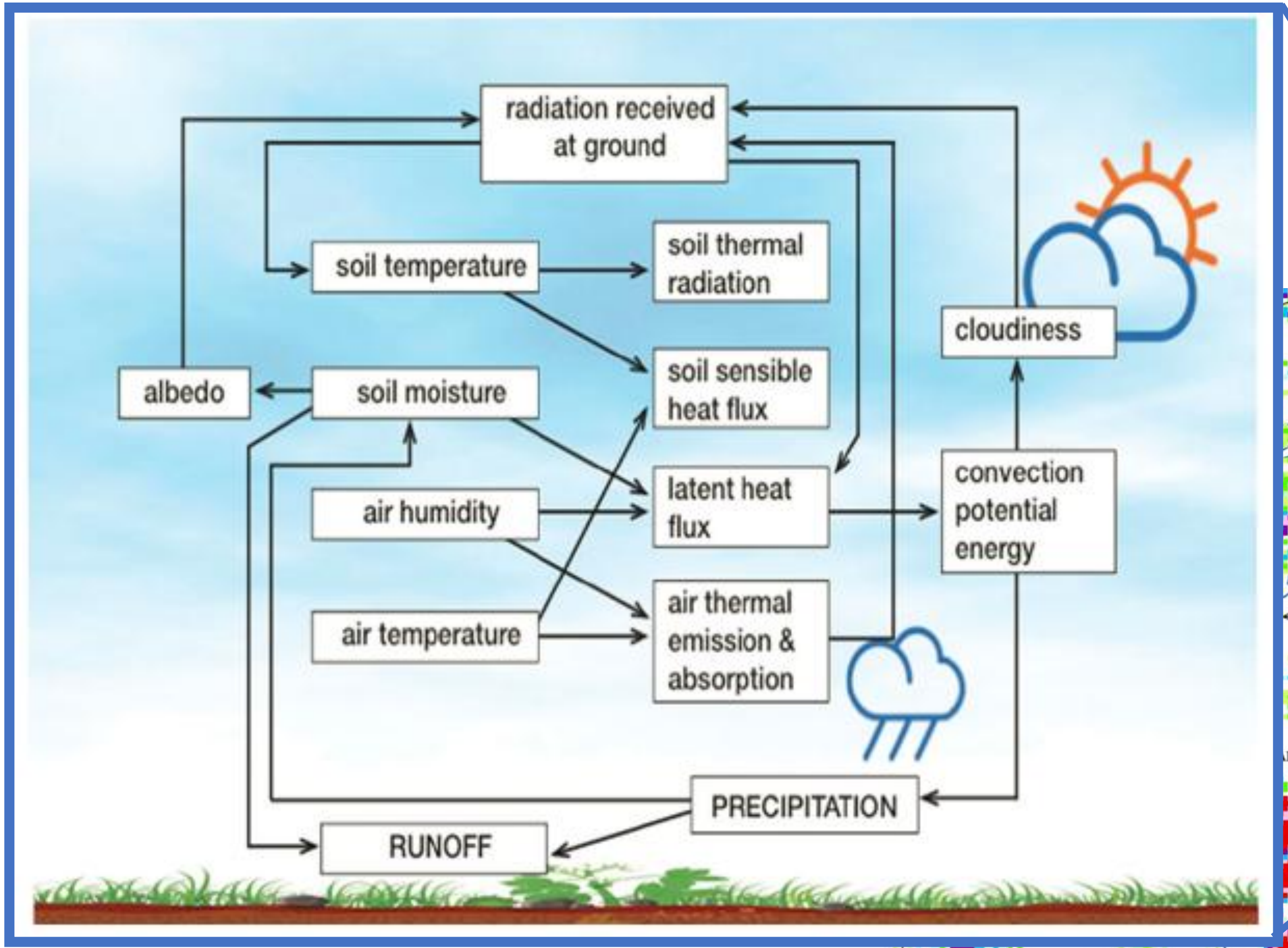
Irrigated Agriculture in a Desert Environment

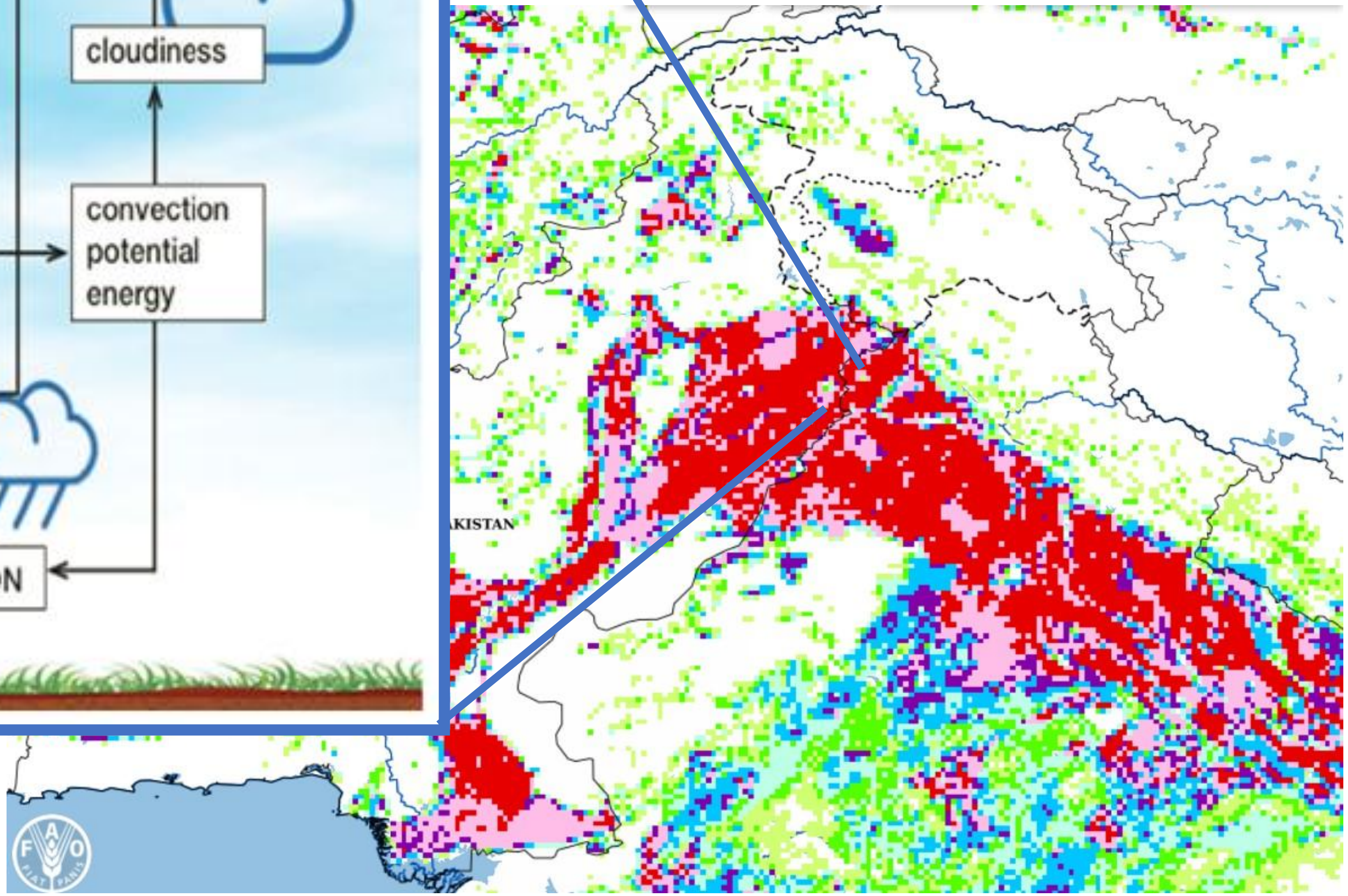
https://theconversation.com/technologies-will-tackle-irrigation-inefficiencies-in-agricultures-drier-future-40601

Irrigation Modifies Earth System Dynamics



https://doi.org/10.1038/s43017-023-00438-5


radiation received at ground
soil temperature
soil thermal radiation
soil sensible heat flux
albedo
soil moisture
latent heat flux
air humidity
air thermal emission & absorption
air temperature
cloudiness
convection potential energy
PRECIPITATION
RUNOFF


AKISTAN

Water-Balance in the Root Zone

Surface Water Balance Model

$$nZ_r \frac{d\theta(t)}{dt} = \varphi(\theta(t), t) - \chi(\theta(t))$$

n soil porosity, Z_r root zone height

- Surface Gains

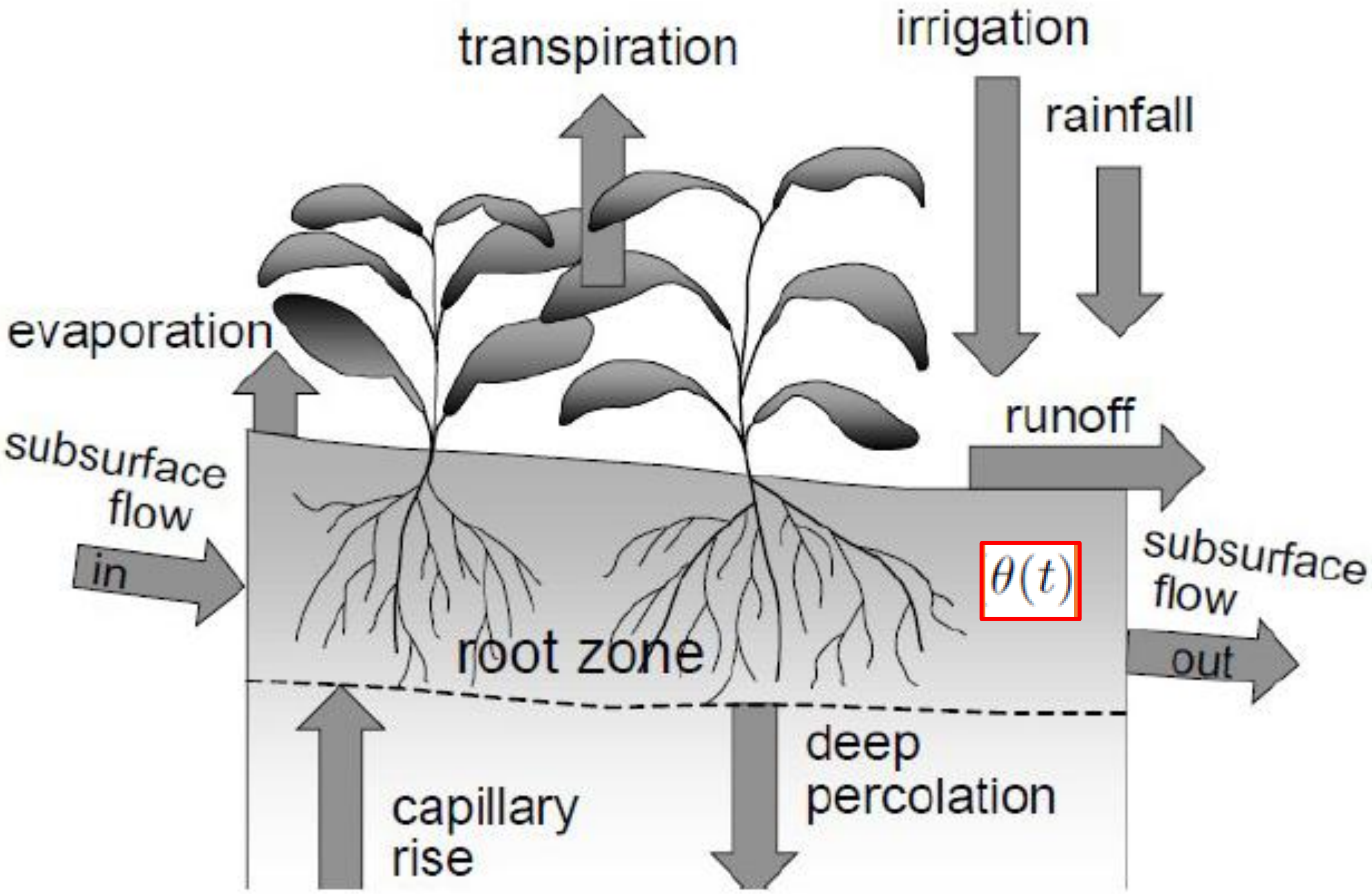
$$\varphi(\theta(t), t) = I_t^g + I_t^s + R(\theta(t), t)$$

I irrigation, $R(\theta(t), t)$ effective rainfall rate

- Surface Losses

$$\chi(\theta(t)) = E(\theta(t)) + L(\theta(t))$$

Losses due to Evapotranspiration $E(\theta(t))$ and deep percolation $L(\theta(t))$



$$nZ_r \frac{d\theta(t)}{dt} = \varphi(\theta(t), t) - \chi(\theta(t))$$

Energy Balance

Evaporation + Transpiration = Evapotranspiration

Surface Energy Balance (SEB) Models

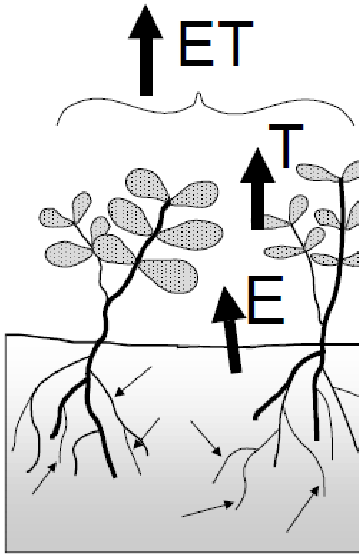
$\lambda ET = Rn - G - H$

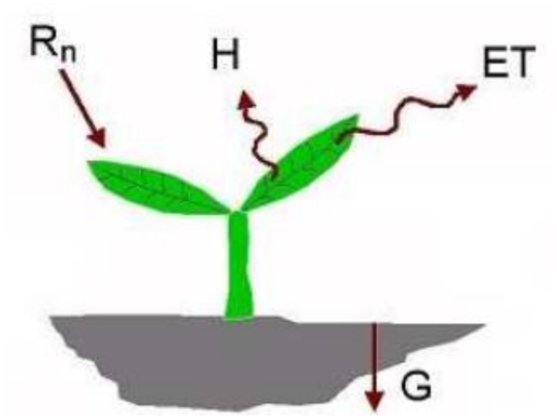
λET - Residual/ latent heat flux (W/m^2)

R_n - Net incoming radiation (W/m^2)

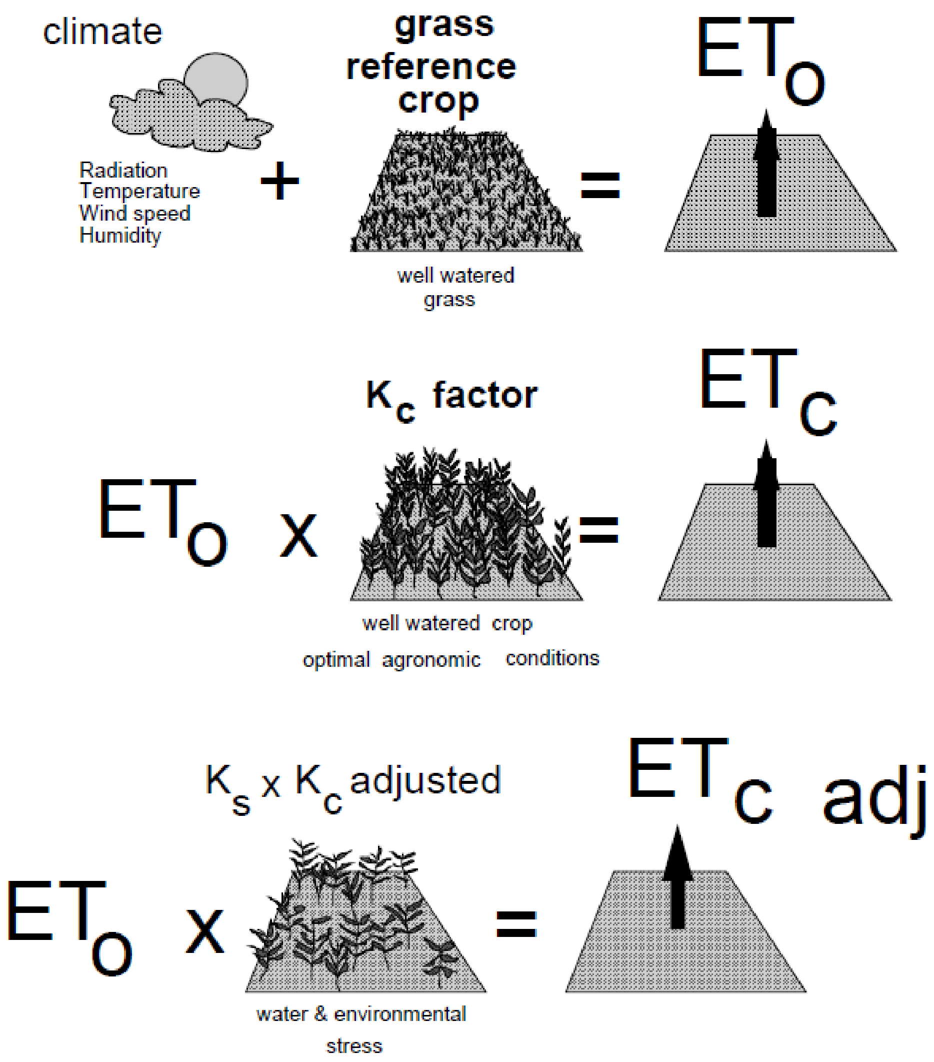
G - Soil Heat flux (W/m^2)

H - Sensible heat flux (W/m^2)

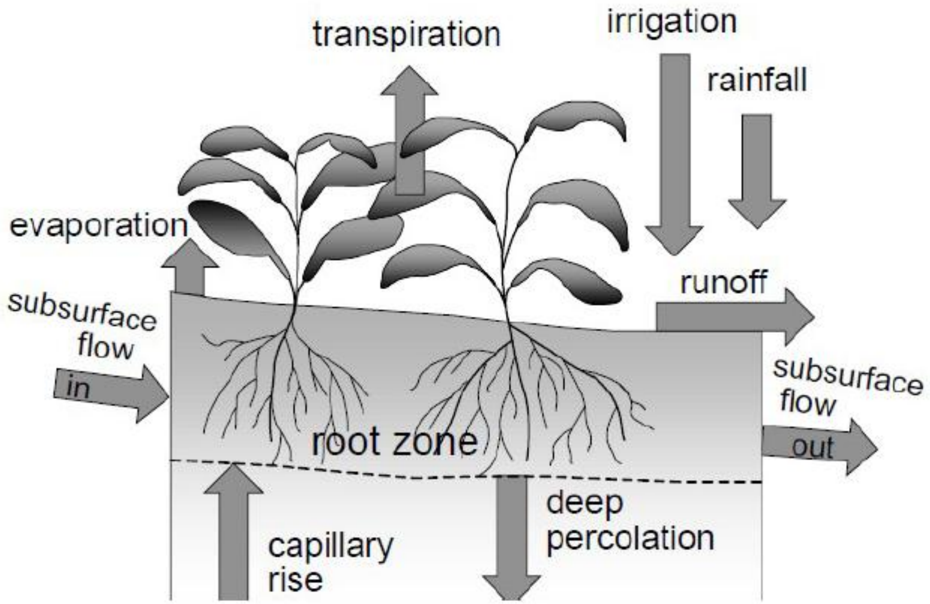




$$\lambda ET = Rn - G - H$$



Leakage to Groundwater



Deep Percolation: Gravity-driven water leakages below the crop rootzone

$$L(\theta(t)) = K_s \left(\frac{\theta(t) - \theta_{hs}}{\theta_s - \theta_{hs}} \right)^{3+\left(\frac{2}{\lambda}\right)}$$

θ_s– soil moisture at wilting point
K_s– saturated hydraulic conductivity (m/s or cm/day)
λ– pore sizes distribution index

θ_{hs}– soil moisture at field capacity
β– model parameter

Losses are maximum when the soil is fully saturated.

Hydraulic conductivity drops sharply as soil dries

Brocca, L., F. Melone, and T. Moramarco. "On the estimation of antecedent wetness conditions in rainfall–runoff modelling." *Hydrological Processes: An International Journal* 22.5 (2008)

Laio, Francesco, et al. "Plants in water-controlled ecosystems: active role in hydrologic processes and response to water stress: II. Probabilistic soil moisture dynamics." Advances in water resources 24.7 (2001)

Note:

- Primary challenges related to Earth Systems Dynamics
  - They are highly nonlinear.
  - They are extremely high-dimensional.
  - They are mostly unknown.

- If we discover ESD, we use them for:
  - Interpretation
  - Predictions / forecasts
  - Optimization & Control

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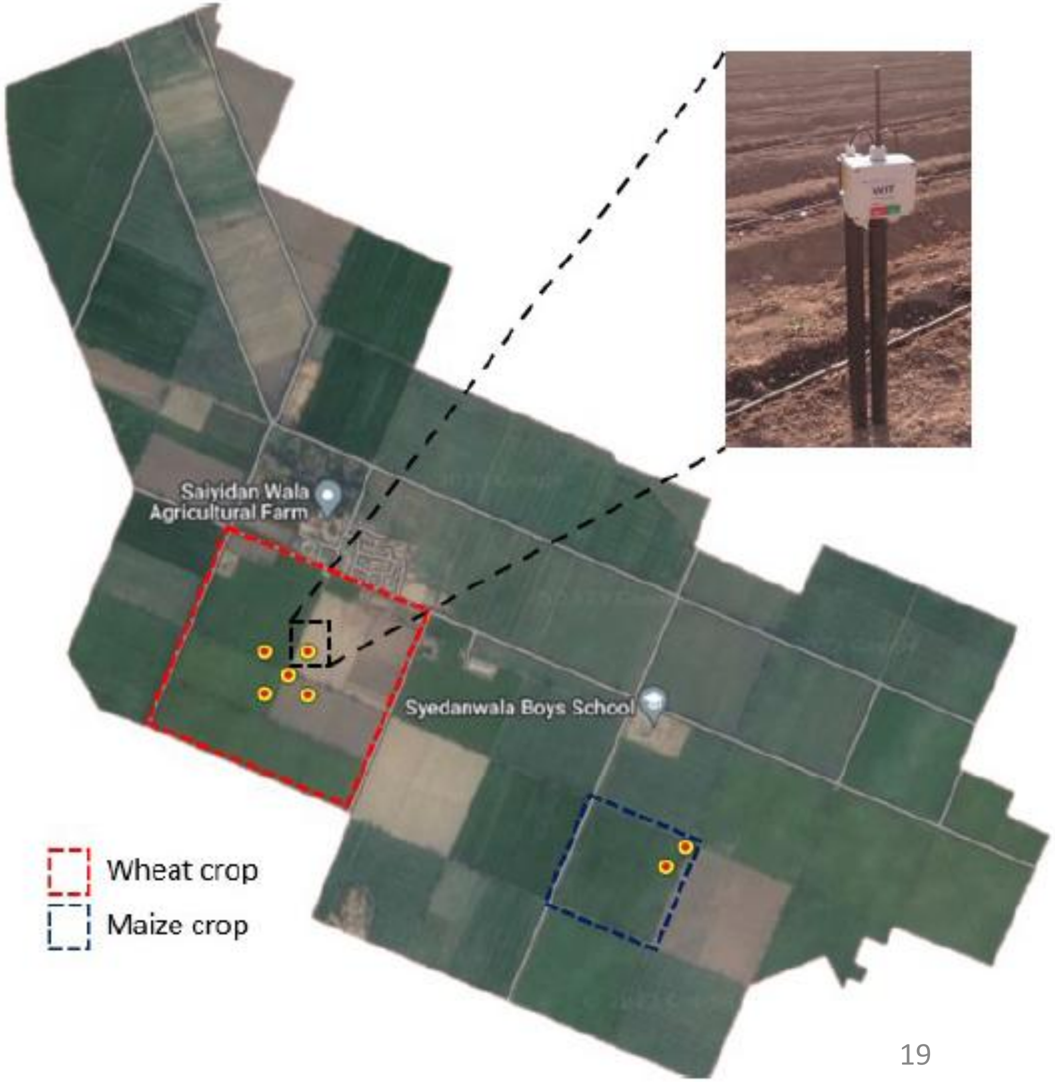
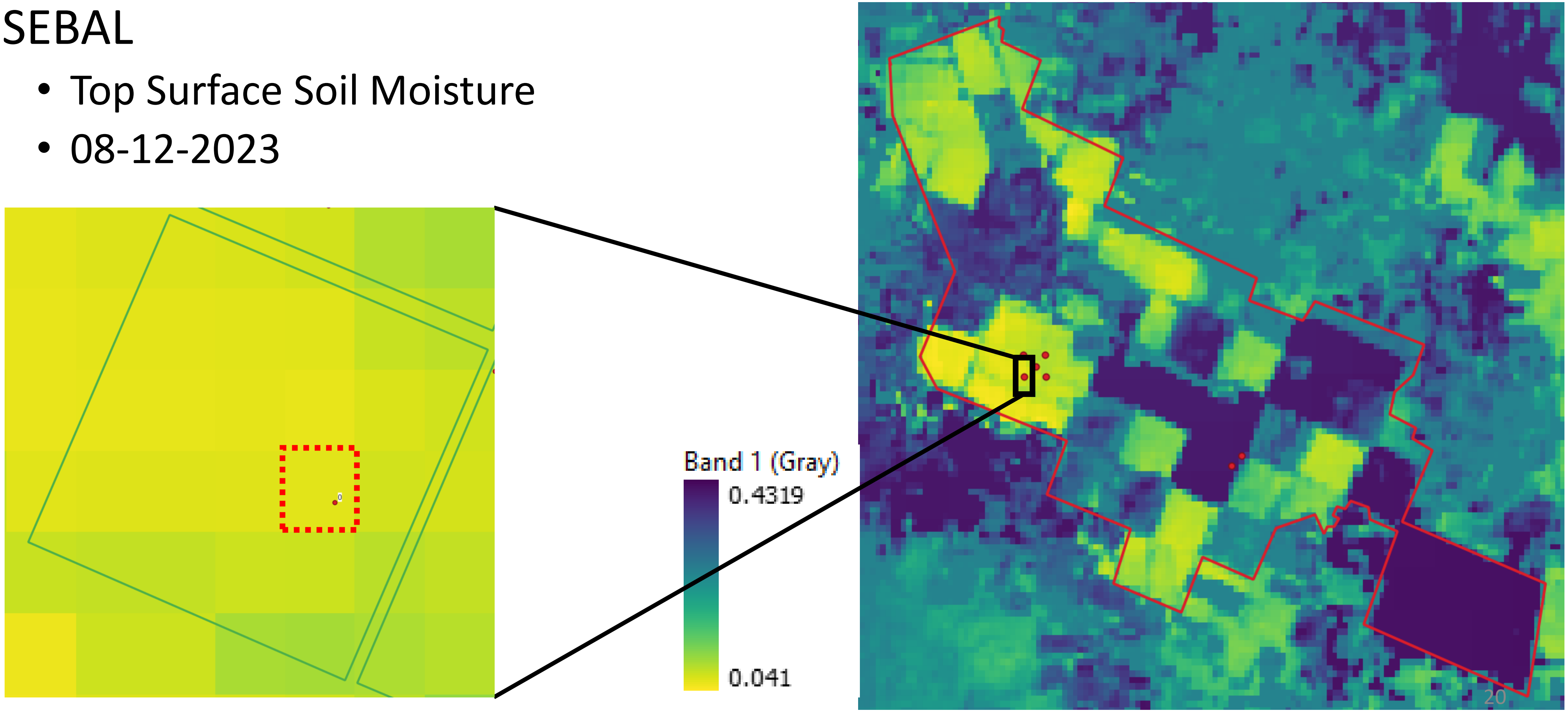
Example: Soil Moisture Modeling

- 1200 Acres / 485 hectares
- Rabi Season
  - 1 December: 23 – 30 April, 24
- 100 acres Wheat
- 25 acres subplots

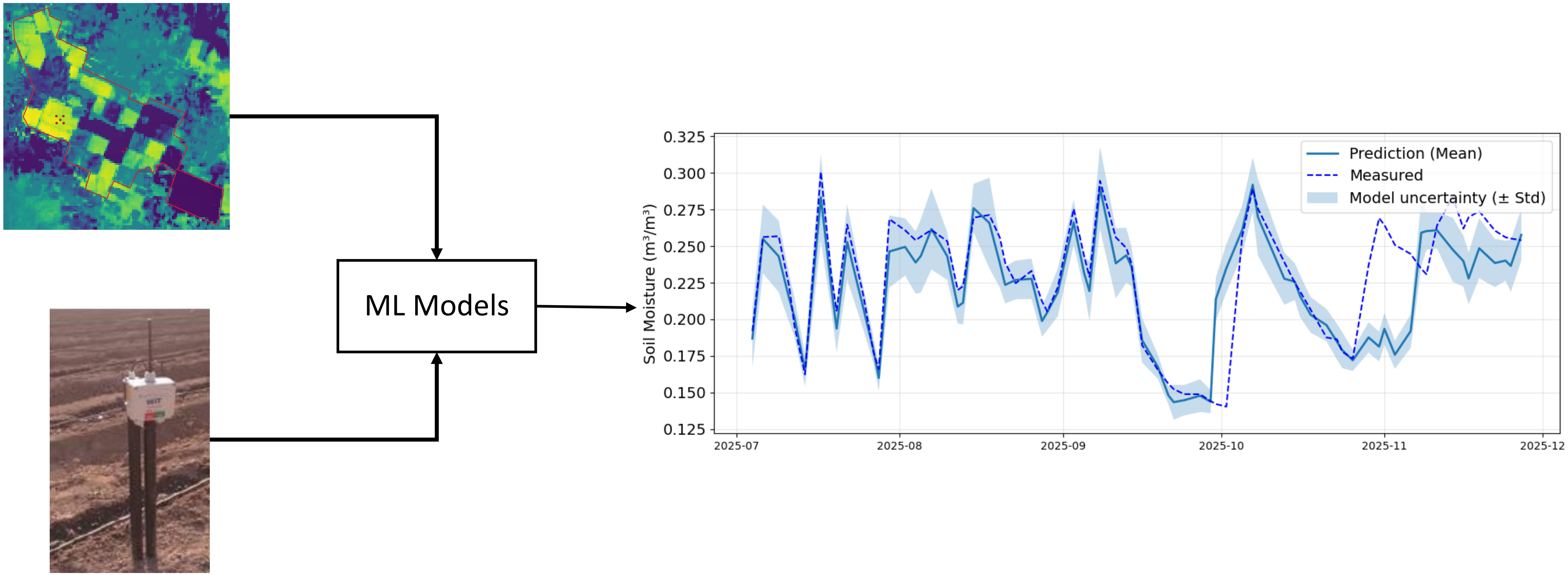


SEBAL Soil Moisture Estimates

- SEBAL
  - Top Surface Soil Moisture
  - 08-12-2023

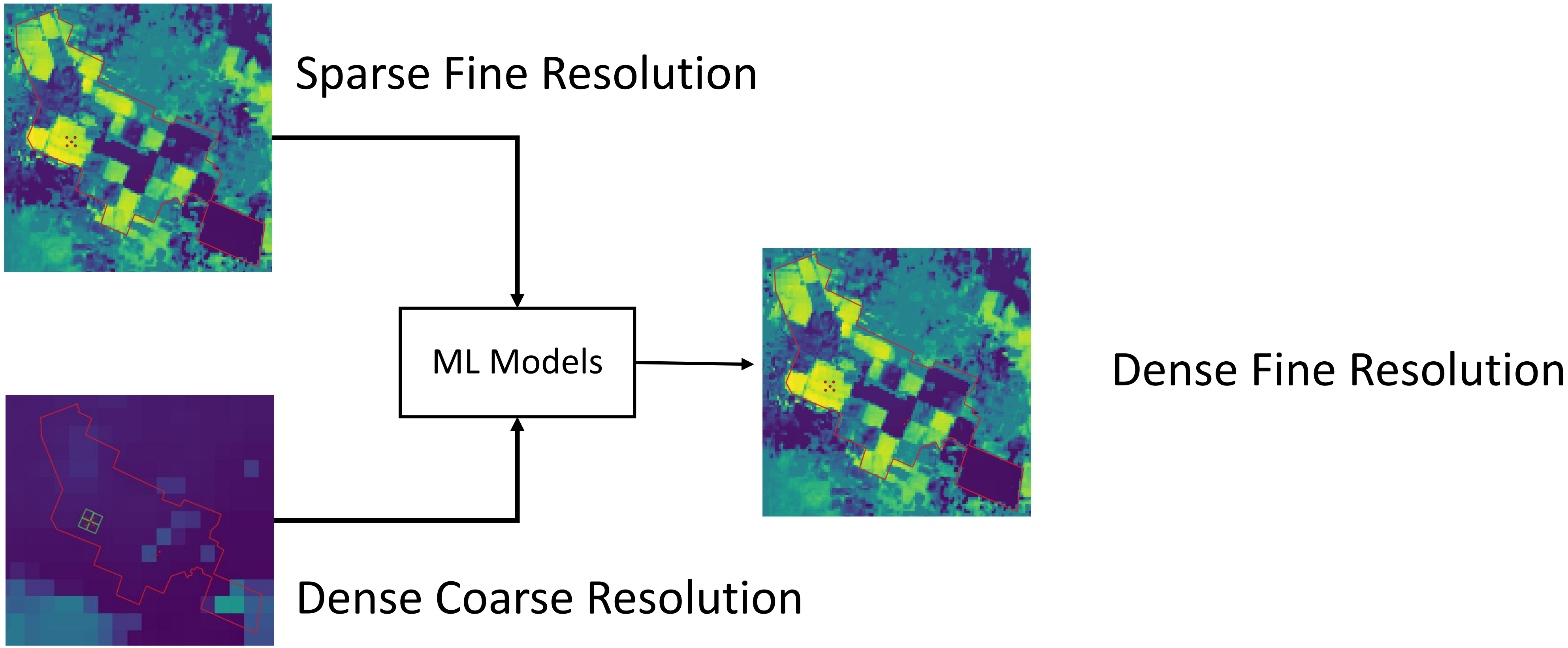


Satellite-based Soil Moisture Temporal Mapping*

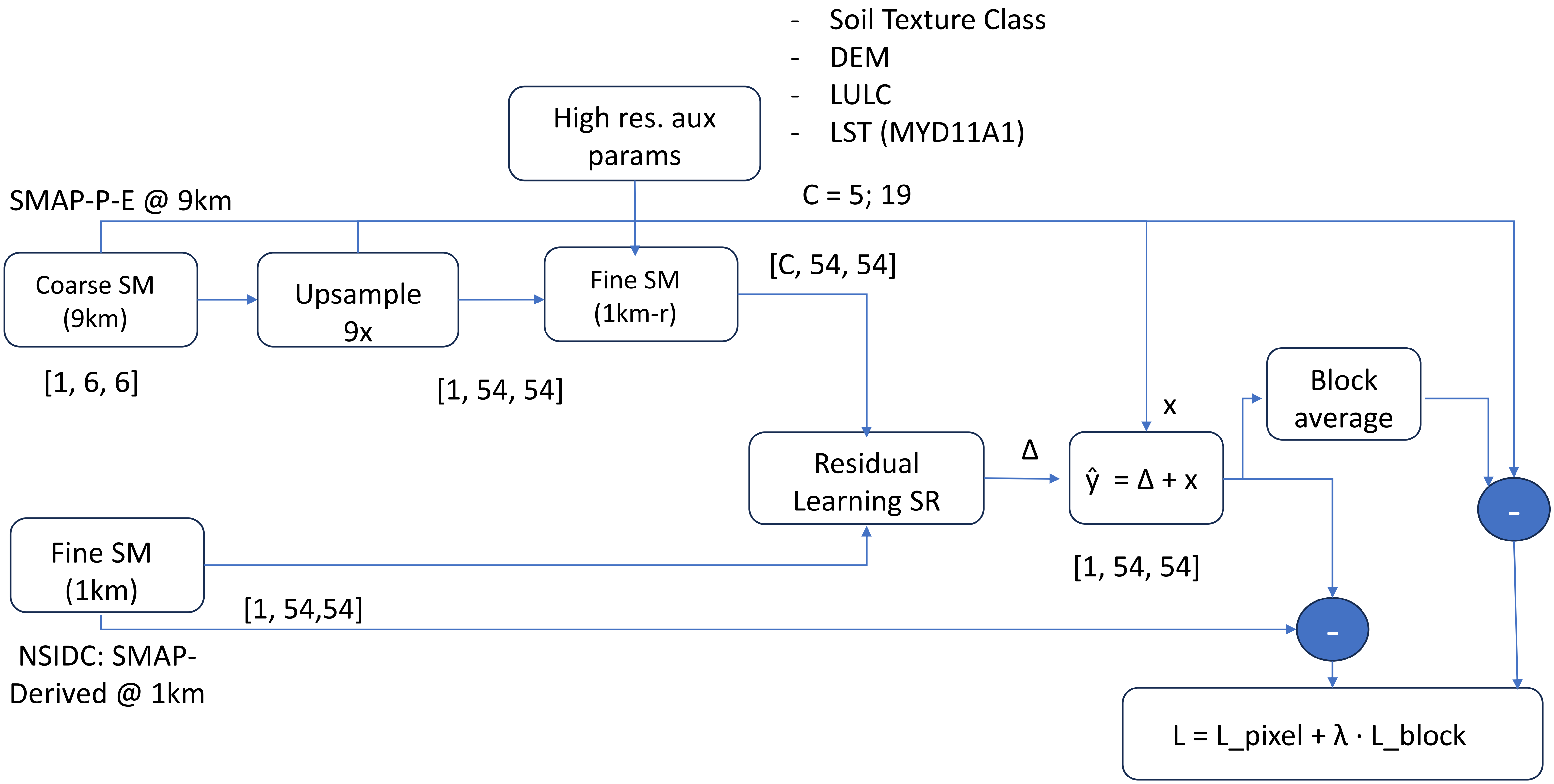


*Accepted at
H. Rafique and A. Muhammad, "A deep learning framework for Farm-scale soil moisture retrievals: A case study for a data-scarce Region," *IGARSS 2026 - 2026 IEEE International Geoscience and Remote Sensing Symposium*, Washington, D.C., 2026.

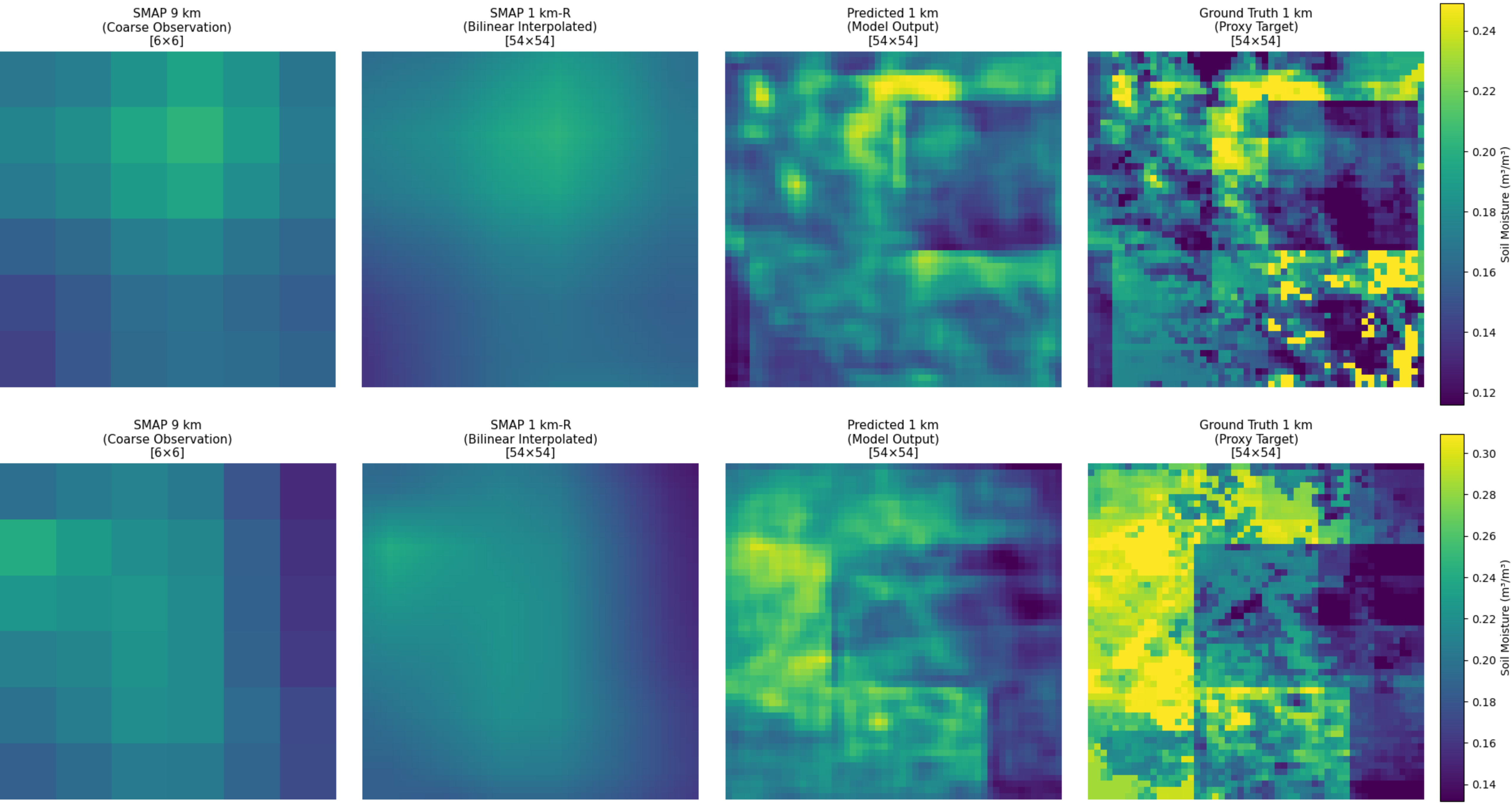
Satellite-based Soil Moisture Spatial Mapping*



*Under Review in
H. Rafique, A. Muhammad, and Jawairia Ishfaq, "Thermal Guided Super Resolution for Real-time Downscaling SMAP Soil Moisture," in IEEE Geoscience and Remote Sensing Letters, 2026.



Two Aqua MODIS products at 1-km resolution—daily daytime/nighttime LST (MYD11A1) and biweekly NDVI (MYD13A2)


SMAP 9 km
(Coarse Observation)
[6×6]
SMAP 1 km-R
(Bilinear Interpolated)
[54×54]
Predicted 1 km
(Model Output)
[54×54]
Ground Truth 1 km
(Proxy Target)
[54×54]
0.24
0.22
0.20
0.18
0.16
0.14
0.12
Soil Moisture (m³/m³)
SMAP 9 km
(Coarse Observation)
[6×6]
SMAP 1 km-R
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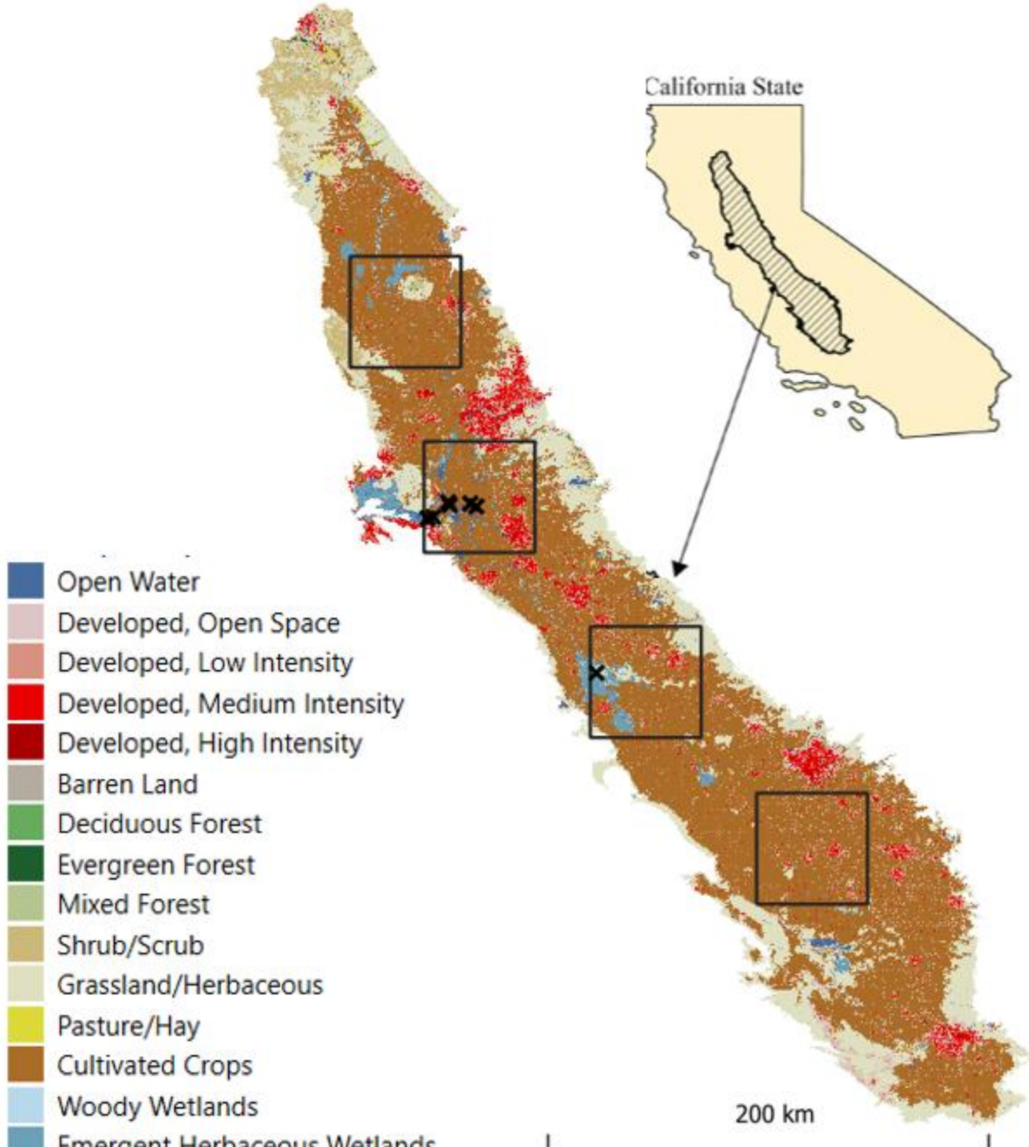
[54×54]
Predicted 1 km
(Model Output)
[54×54]
Ground Truth 1 km
(Proxy Target)
[54×54]
0.30
0.28
0.26
0.24
0.22
0.20
0.18
0.16
0.14
Soil Moisture (m³/m³)

Training Data

Cloud cover pixels:
- 9km < 2 pixels
- 1km < 25%

- Samples Extraction
  - 9km
    - 6 x 6 pixels
  - 1km
    - 56 x 56 pixels
- Train data: ~ 5.5 years
  - F: 04/2015 to 12/2021 : 1117 samples
  - B: 01/2017 to 06/2024: 1063
- Test data: 1.5 year
  - F: 01/2022 to 06/2024: 210
  - B: 04/2015 to 12/2021: 264



Results

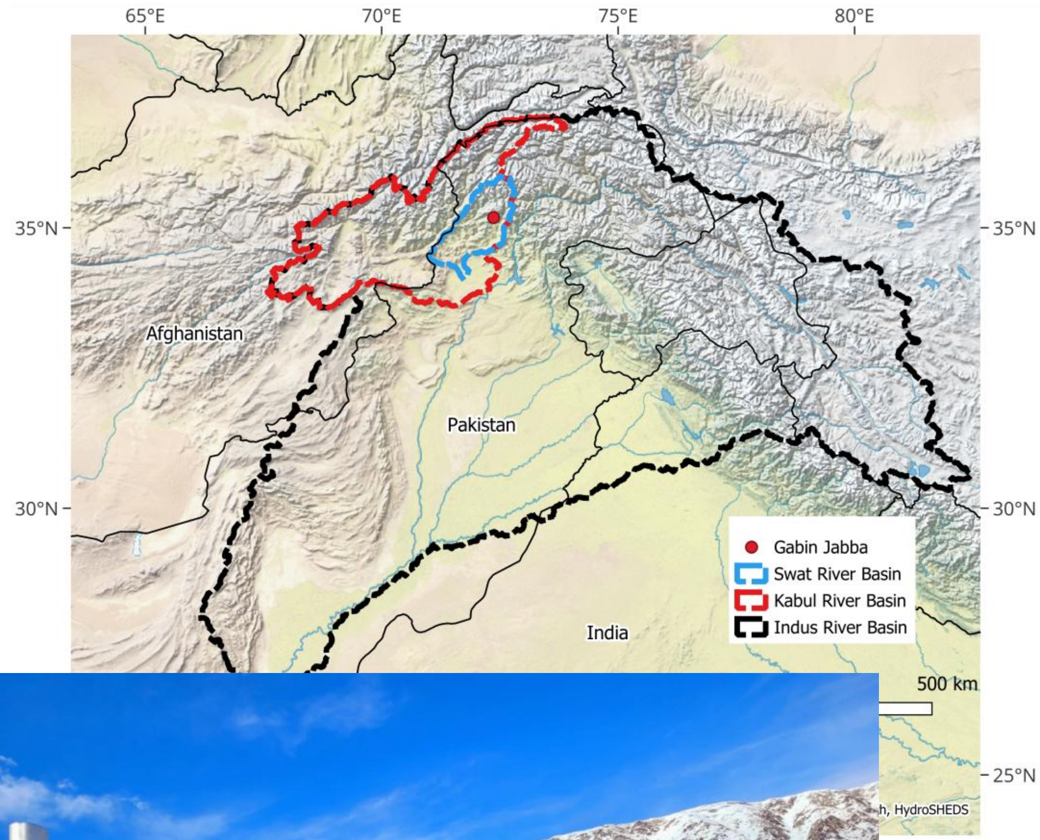
Model	UbRMSE	Bias	MSE	PSNR (dB)	PearsonR
Baseline-forward	0.0382	-0.0076	0.0017	16.3001	0.5817
SRCNN-forward	0.0261	-0.0033	0.0008	19.5081	0.8581

Model	UbRMSE	Bias	MSE	PSNR (dB)	PearsonR
Baseline-backward	0.0355	-0.0103	0.0019	14.4425	0.5295
SRCNN-backward	0.0216	-0.0049	0.0008	18.7526	0.8813

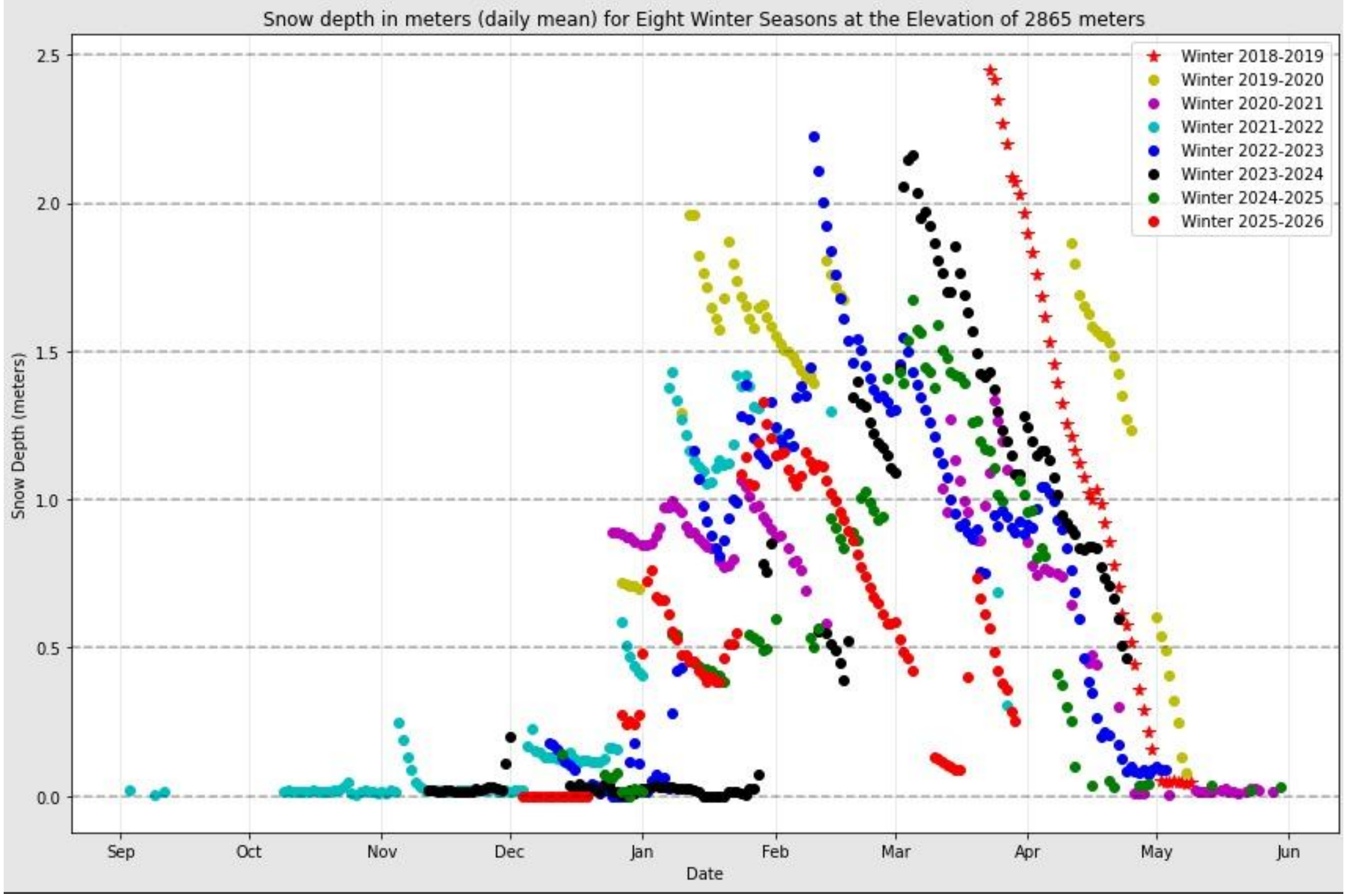
Model	UbRMSE	Bias	MSE	PSNR (dB)	PearsonR
Baseline-mean	0.03685	-0.00895	0.0018	15.37	0.5559
SRCNN-mean	0.02385	-0.00410	0.0008	19.13	0.8700

Snow Depth Gap Filling

- Snow depth : a proxy for water availability later in the season.
- Decadal trends show impacts of climate change.
- **Problem:** Communication and sensor problems lead to gaps in data.
- **Solution:** Use a Random Forest model trained on the available snow depth data as well as predictors from a uniform and continuous data source (ERA5-Land[1]) to predict snow depth values for days where sensor was not working.
- Model able to predict with mean absolute error of 0.09 m.

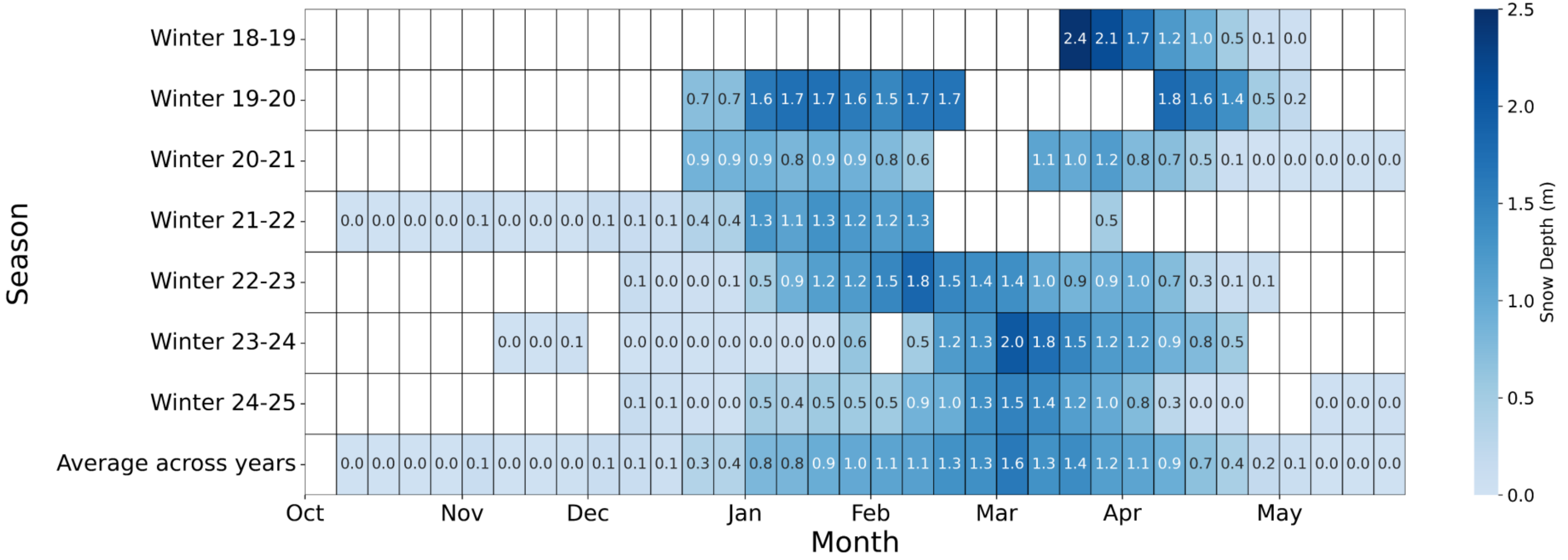


1. https://confluence.ecmwf.int/display/CKB/ERA5-Land%3A+data+documentation

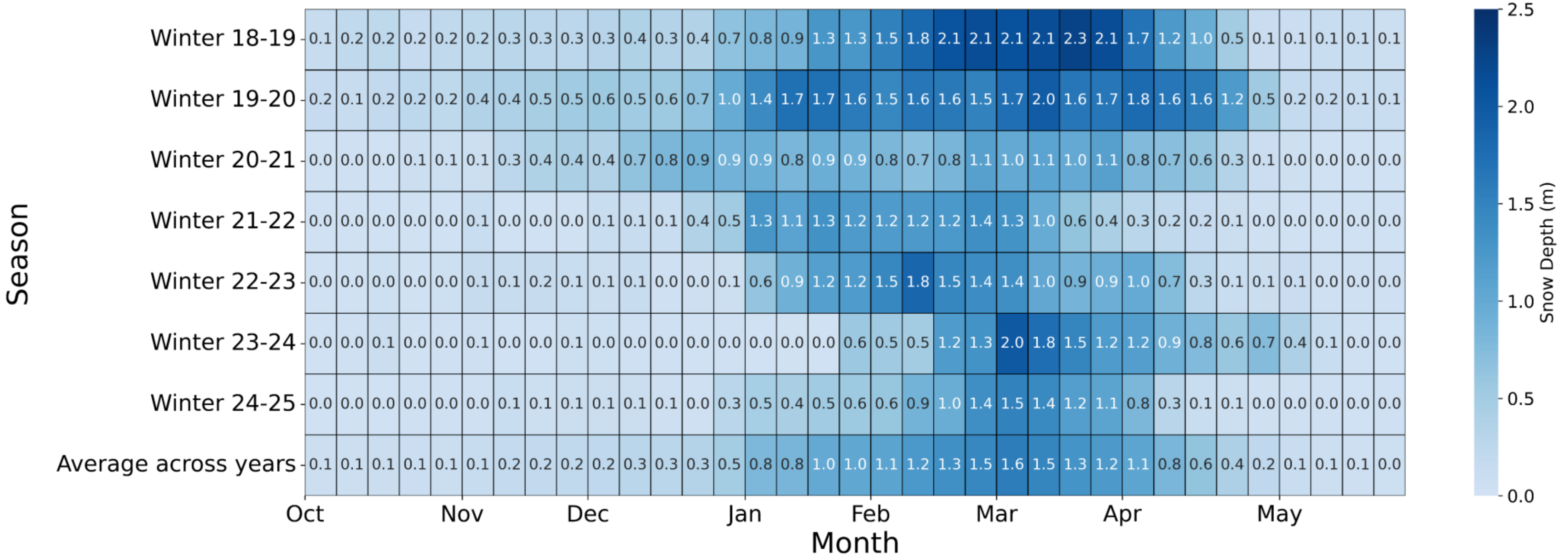


Snow depth in meters (daily mean) for Eight Winter Seasons at the Elevation of 2865 meters
Winter 2018-2019
Winter 2019-2020
Winter 2020-2021
Winter 2021-2022
Winter 2022-2023
Winter 2023-2024
Winter 2024-2025
Winter 2025-2026
Snow Depth (meters)
2.5
2.0
1.5
1.0
0.5
0.0
Sep
Oct
Nov
Dec
Jan
Feb
Mar
Apr
May
Jun
Date

Original Data



Gap Filled Data



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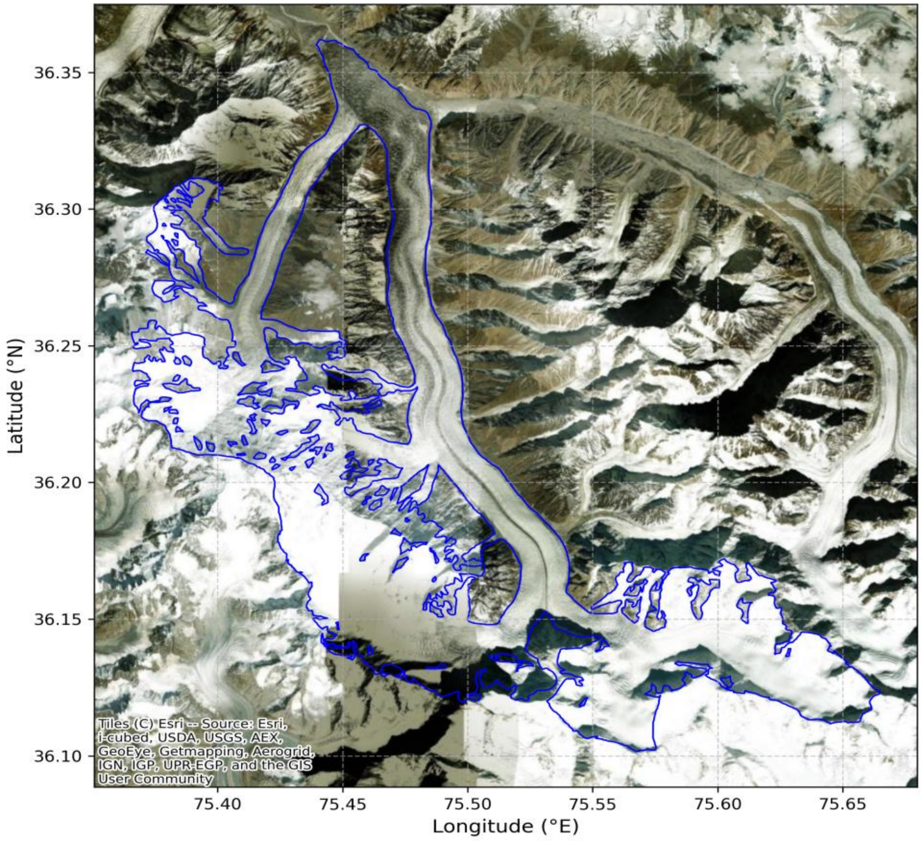
Early Warning Signals in Earth System Dynamics

- Many Earth systems (glaciers, climate, ecosystems) exhibit tipping behavior
- Sudden transitions can have catastrophic consequences
- Detecting Early warning signals (EWS) is critical for risk mitigation

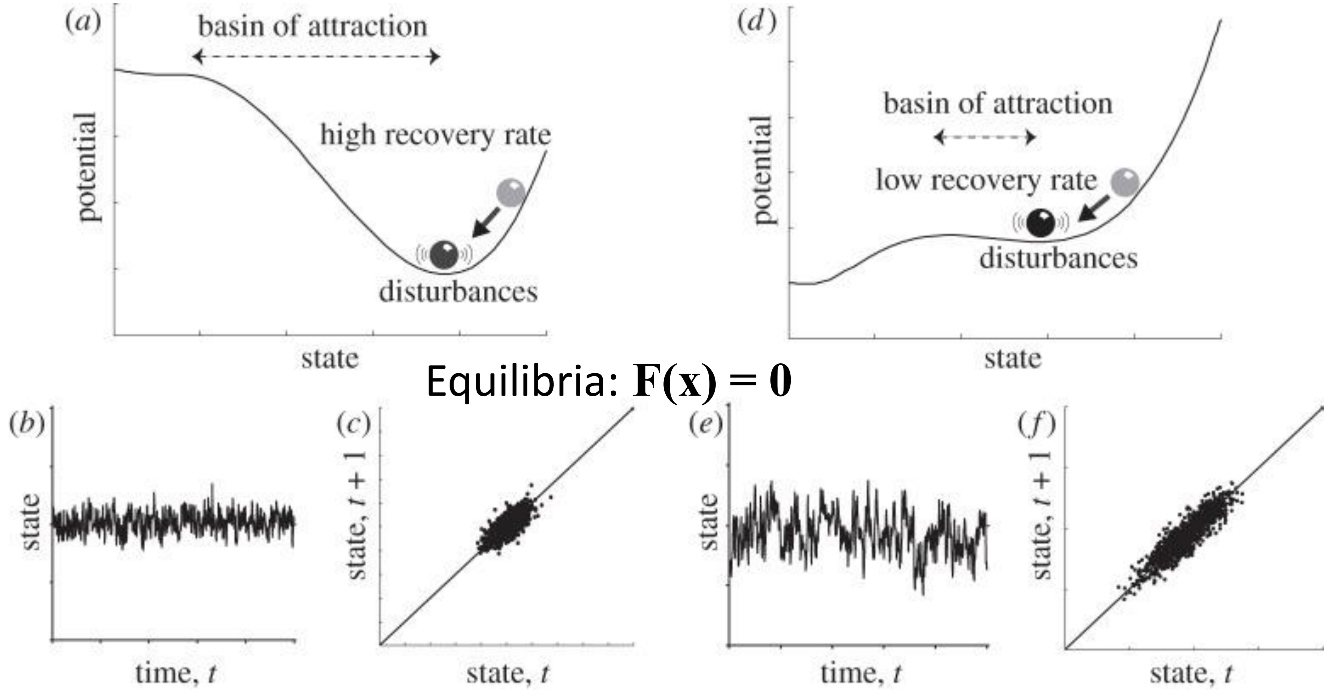
- Khurdopin: Large surge-type glacier (approx. 40 km length) in Shimshal Valley, Karakoram, Pakistan
- Surged in 2017 blocking the Shimshal river
- Upcoming surge predicted a year in advance

Steiner, J.F. *et al.* (2018) *The Khurdopin Glacier Surge Revisited – extreme flow velocities and formation of a dammed lake in 2017, The Cryosphere*.



Critical Slowing Down as an Early Warning Signal

- Near a tipping point, system stability gradually weakens
- Small disturbances persist longer instead of decaying quickly
- Observable effects:
  - Increased autocorrelation
  - Increased variance



$$\mathbf{x}_{n+1} = \mathbf{F}(\mathbf{x}_n)$$

Lenton, T.M. et al. (2012) *Early warning of climate tipping points from critical slowing down: Comparing methods to improve robustness, Philosophical Transactions of the Royal Society*

But we don't know the Dynamics!

$$\mathbf{x}_{n+1} = \mathbf{F}(\mathbf{x}_n)$$

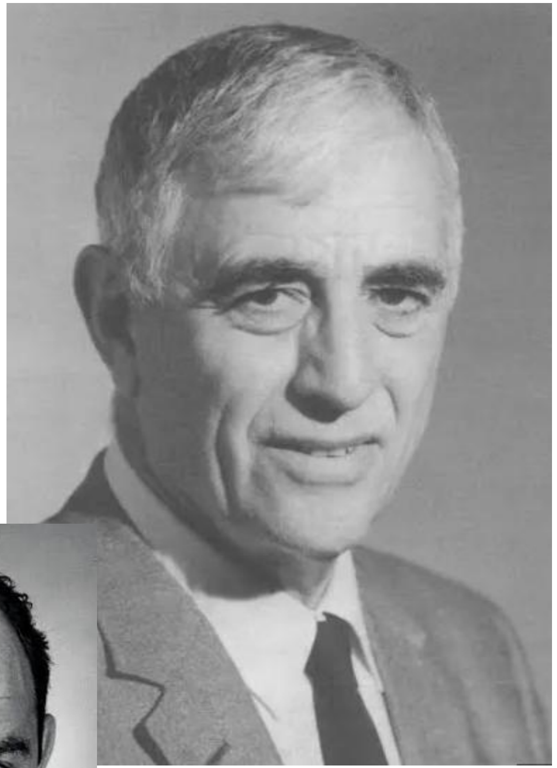
- Measure $\mathbf{X}$ by direct measurements, state estimation, downscaling etc. (snapshots)

$$\left\{\mathbf{x}^{(m)}, \mathbf{y}^{(m)}\right\}_{m=1}^{M} \quad \text{such that} \quad \mathbf{y}^{(m)} = \mathbf{F}(\mathbf{x}^{(m)}), \quad m = 1, \ldots, M.$$

$$\mathbf{X} = \begin{pmatrix} \mathbf{x}^{(1)} & \mathbf{x}^{(2)} & \cdots & \mathbf{x}^{(M)} \end{pmatrix} \in \mathbb{R}^{d \times M}, \quad \mathbf{Y} = \begin{pmatrix} \mathbf{y}^{(1)} & \mathbf{y}^{(2)} & \cdots & \mathbf{y}^{(M)} \end{pmatrix} \in \mathbb{R}^{d \times M}$$

- How to find $\mathbf{F}$?

Enter Koopman and von Neumann!

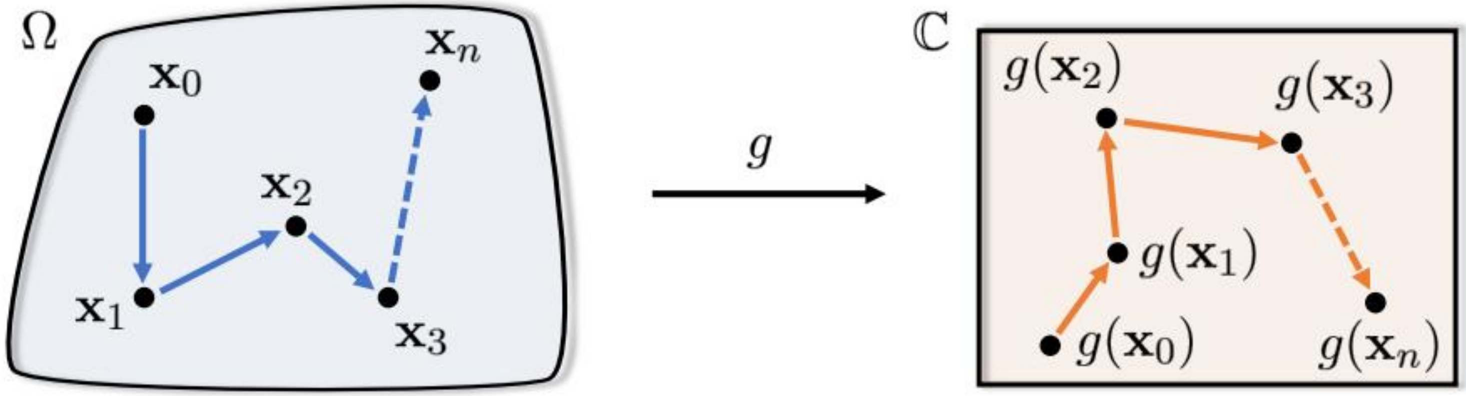


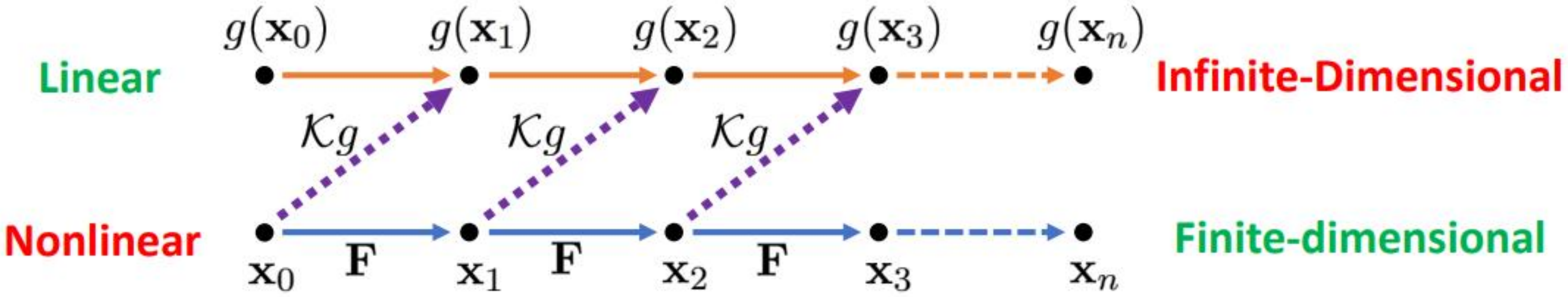
- In 1931, Bernard Koopman (and later with John von Neumann) introduced an operator-theoretic approach to dynamical systems, initially to describe Hamiltonian systems.
- Koopman operators offer a powerful alternative to the classical geometric view of dynamical systems by addressing the fundamental issue of nonlinearity.

Koopman, B. O. (1931), 'Hamiltonian systems and transformation in Hilbert space', *Proceedings of the National Academy of Sciences* **17**(5), 315–318.

Koopman, B. O. & von Neumann, J. (1932), 'Dynamical systems of continuous spectra', *Proceedings of the National Academy of Sciences* **18**(3), 255–263.

Primer on Koopman Theory



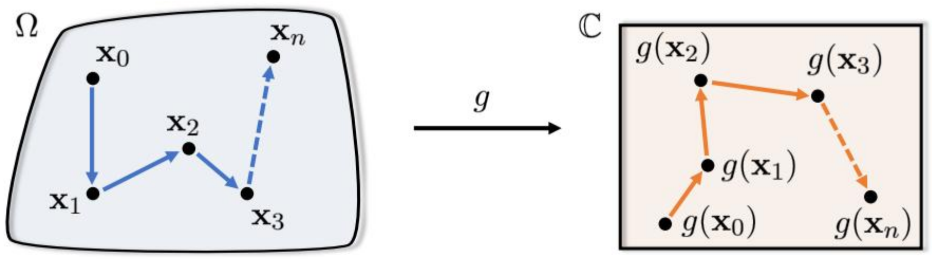


$$[\mathcal{K}g](\mathbf{x}) = g(\mathbf{F}(\mathbf{x})), \quad \text{so that} \quad [\mathcal{K}g](\mathbf{x}_n) = g(\mathbf{x}_{n+1})$$

Think of it as a coordinate transformation, under which even strongly nonlinear dynamics may be approximated by a linear system

Ref: Matthew J. Colbrook, The Multiverse of Dynamic Mode Decomposition Algorithms, 2023.

Primer on Koopman Theory



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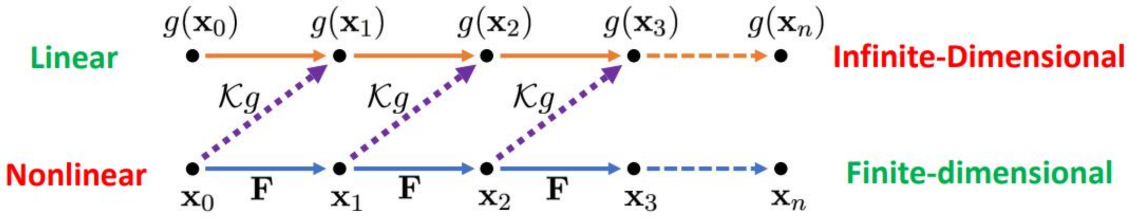

With Linearity, you also get spectra!

If $g \in L^2(\Omega, \omega)$ is an *eigenfunction* of $\mathcal{K}$ with *eigenvalue* λ, then g exhibits perfect coherence

$$g(\mathbf{x}_n) = [\mathcal{K}^n g](\mathbf{x}_0) = \lambda^n g(\mathbf{x}_0) \quad \forall n \in \mathbb{N}.$$

An observable g with $\|g\| = 1$ and $\|(\mathcal{K} - \lambda I)g\| \leq \epsilon$ for $\lambda \in \mathbb{C}$ is known as ϵ-pseudoeigenfunction

$$\|\mathcal{K}^n g - \lambda^n g\| \lesssim \mathcal{O}(n\epsilon)$$

$$\mathcal{K}^n g = \sum_j \lambda_j^n \langle g, \phi_j \rangle \phi_j \quad \forall g \in \mathcal{H}_{\mathrm{pp}}, n \in \mathbb{N}.$$

Ref: Matthew J. Colbrook, The Multiverse of Dynamic Mode Decomposition Algorithms, 2023.

Dynamic Mode Decomposition (Mezic, 2003)

- A computational algorithm for constructing K from state snapshots.

$$\mathbf{x}_{n+1} = \mathbf{F}(\mathbf{x}_n)$$

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Solve $\min_{\mathbf{K}_{\mathrm{DMD}} \in \mathbb{C}^{d \times d}} \|\mathbf{Y} - \mathbf{K}_{\mathrm{DMD}} \mathbf{X}\|_{\mathrm{F}}$

to get: $\mathbf{K}_{\mathrm{DMD}} = \mathbf{Y}\mathbf{X}^{\dagger} \in \mathbb{C}^{d \times d}$

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Solve

$$\min_{\mathbf{K}_{\mathrm{DMD}} \in \mathbb{C}^{d \times d}} \|\mathbf{Y} - \mathbf{K}_{\mathrm{DMD}} \mathbf{X}\|_{\mathrm{F}}$$

to get:

$$\mathbf{K}_{\mathrm{DMD}} = \mathbf{Y}\mathbf{X}^{\dagger} \in \mathbb{C}^{d \times d}$$

which can be approx. by SVD:

$$\tilde{\mathbf{K}}_{\mathrm{DMD}} = \mathbf{U}^{*}\mathbf{Y}\mathbf{V}\boldsymbol{\Sigma}^{-1} \in \mathbb{C}^{r \times r}$$

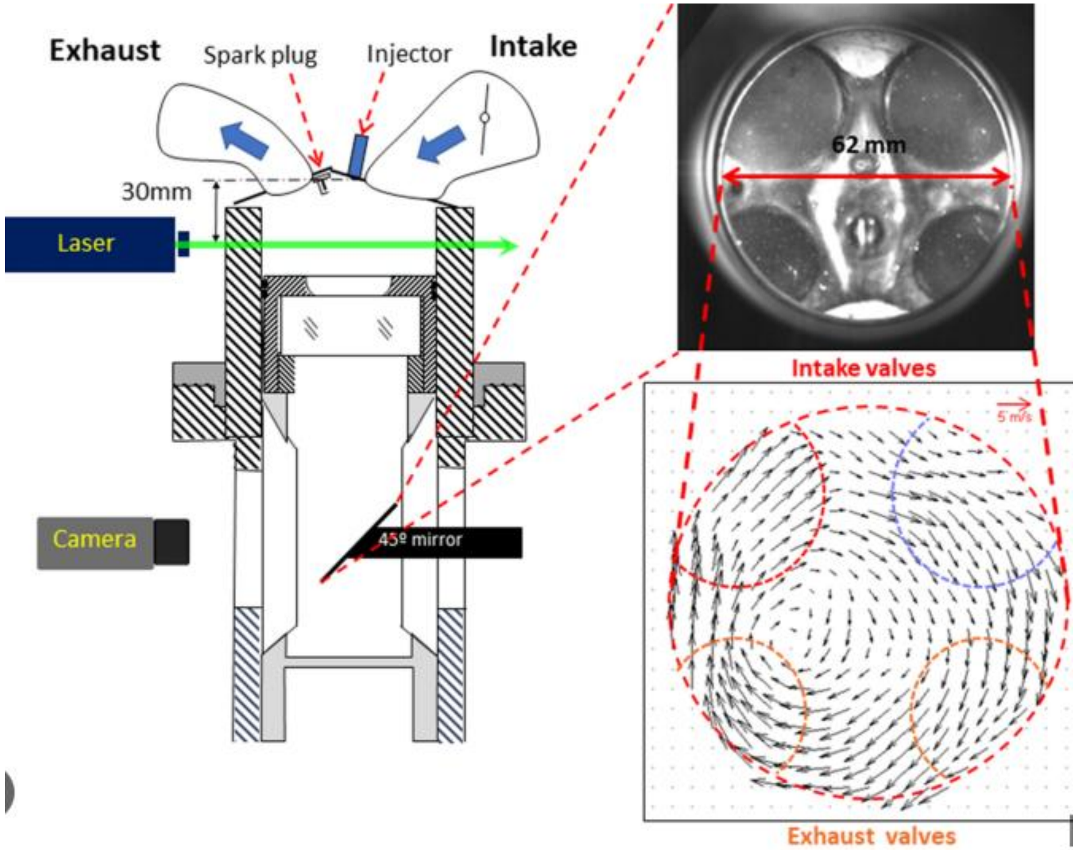
The Galerkin interpretation

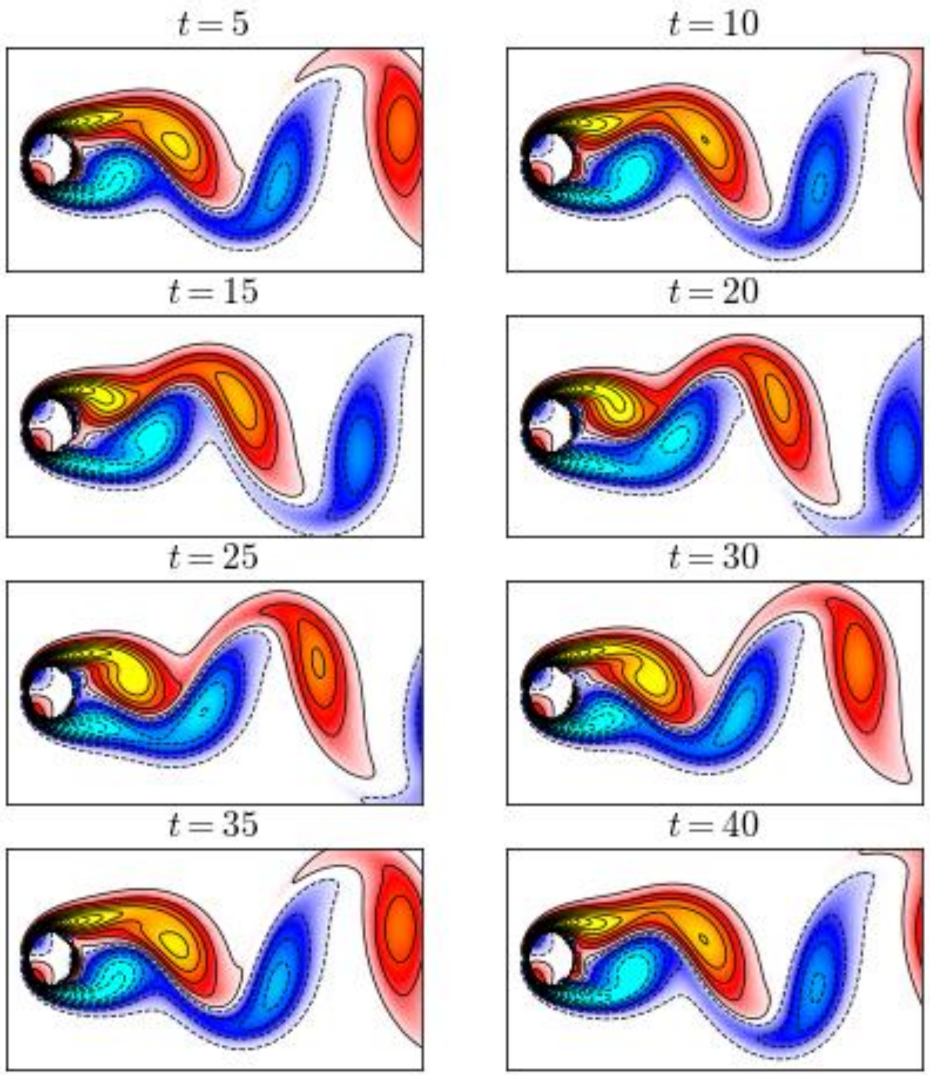
$$[\mathcal{K}g](\mathbf{x}) \approx \sum_{j=1}^{r} u_j(\mathbf{x})(\mathbf{G}^{-1}\mathbf{A}\mathbf{g})_j = \sum_{j=1}^{r} u_j(\mathbf{x})(\tilde{\mathbf{K}}_{\mathrm{DMD}}^{\top}\mathbf{g})_j$$

Time steps: $\mathcal{L}g = \lim_{\Delta t \downarrow 0} \frac{\mathcal{K}_{\Delta t} g - g}{\Delta t}$

$$\mathcal{K}_{\Delta t} = \exp(\Delta t \mathcal{L})$$

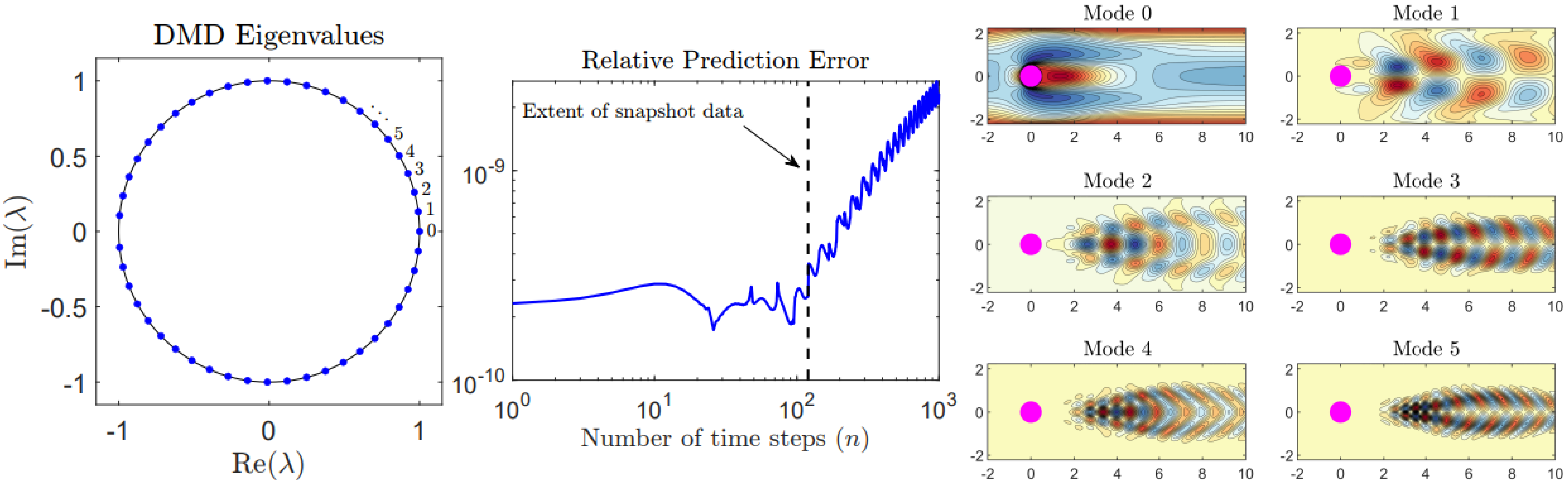
Fluid flow past a cylindrical obstacle





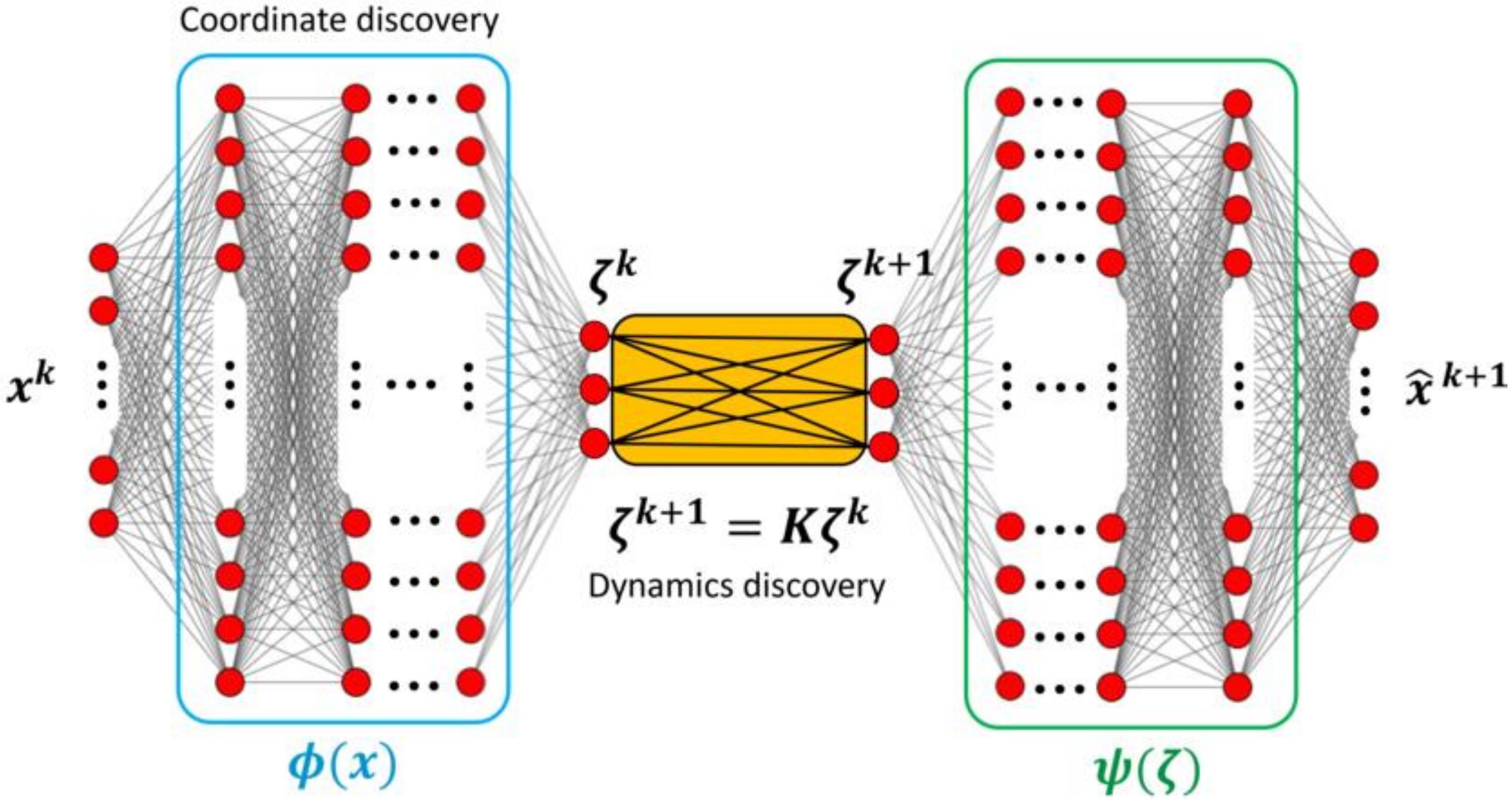
Ref: Matthew J. Colbrook, The Multiverse of Dynamic Mode Decomposition Algorithms, 2023.

Fluid flow past a cylindrical obstacle



Ref: Matthew J. Colbrook, The Multiverse of Dynamic Mode Decomposition Algorithms, 2023.

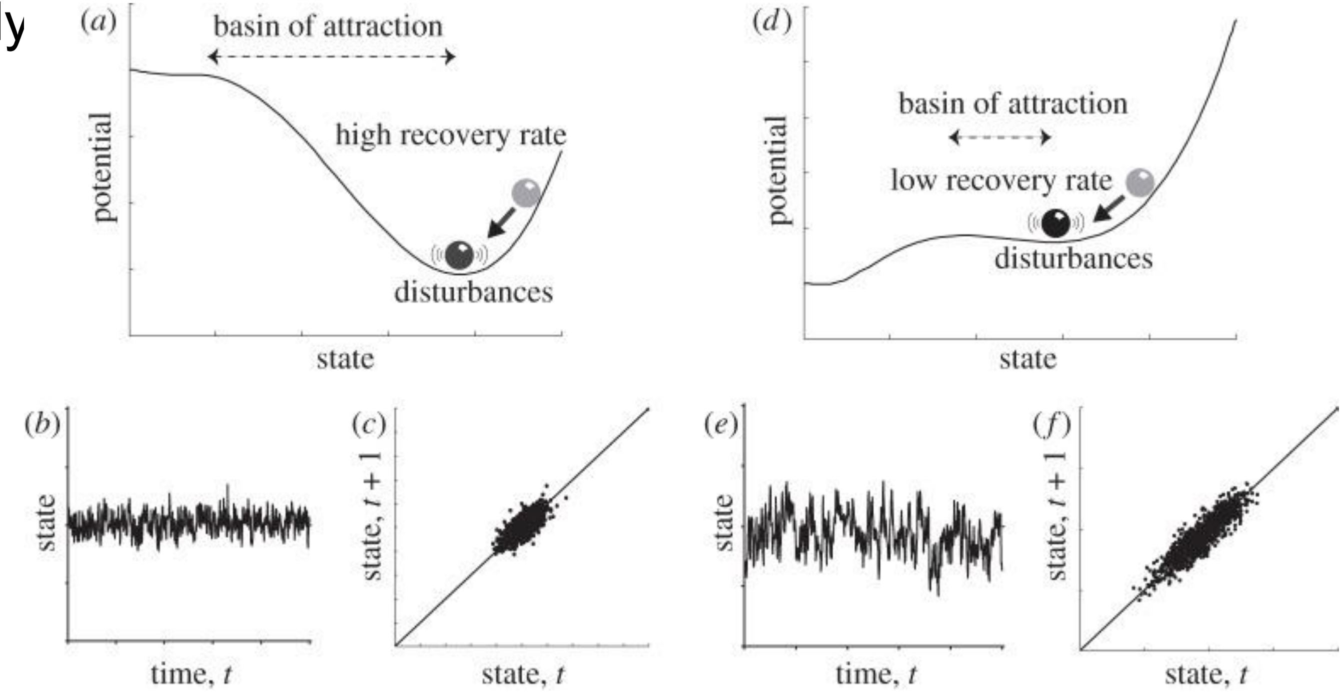
Nonlinearity and High Dimensionality in One-Go



Reza, Maryam, et al. "Progress and future directions on predictive data-driven reduced-order modeling for Electric Propulsion Digital Twins." *Proceedings of the 38th International Electric Propulsion Conference (IEPC 2024), Toulouse, France, June*. 2024.

Critical Slowing Down as an Early Warning Signal

- Near a tipping point, system stability gradually weakens
- Small disturbances persist longer instead of decaying quickly
- Observable effects:
    - Increased autocorrelation
    - Increased variance



- Dynamically, this corresponds to the Dominant (Non-periodic) eigenvalue magnitude approaching 1

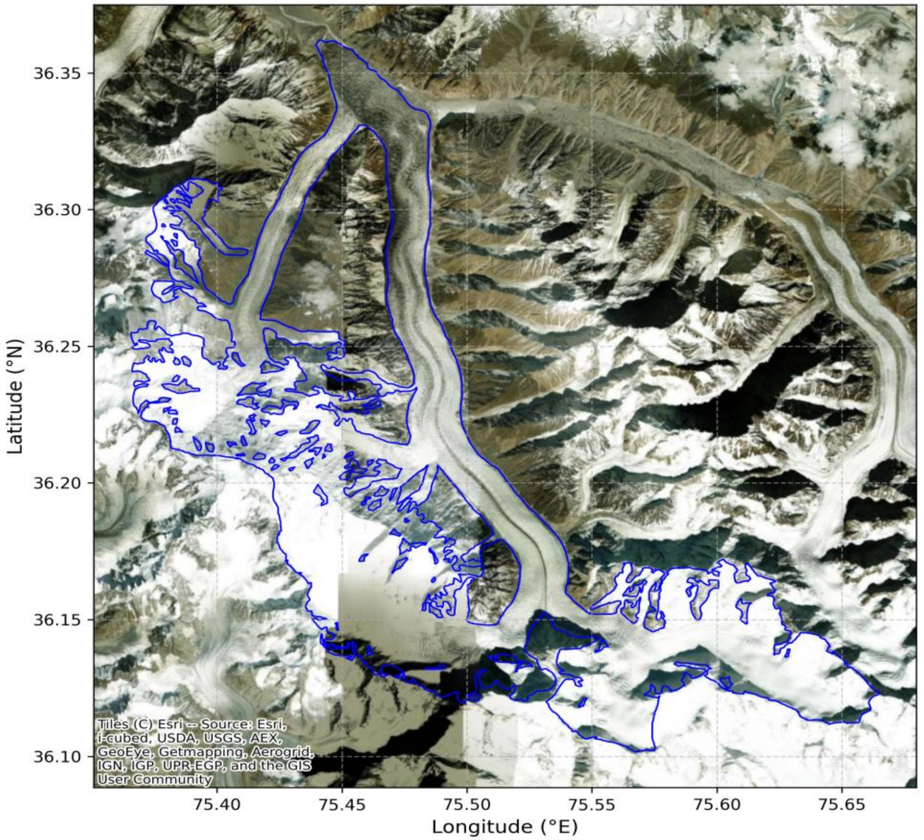
Lenton, T.M. et al. (2012) *Early warning of climate tipping points from critical slowing down: Comparing methods to improve robustness, Philosophical Transactions of the Royal Society*

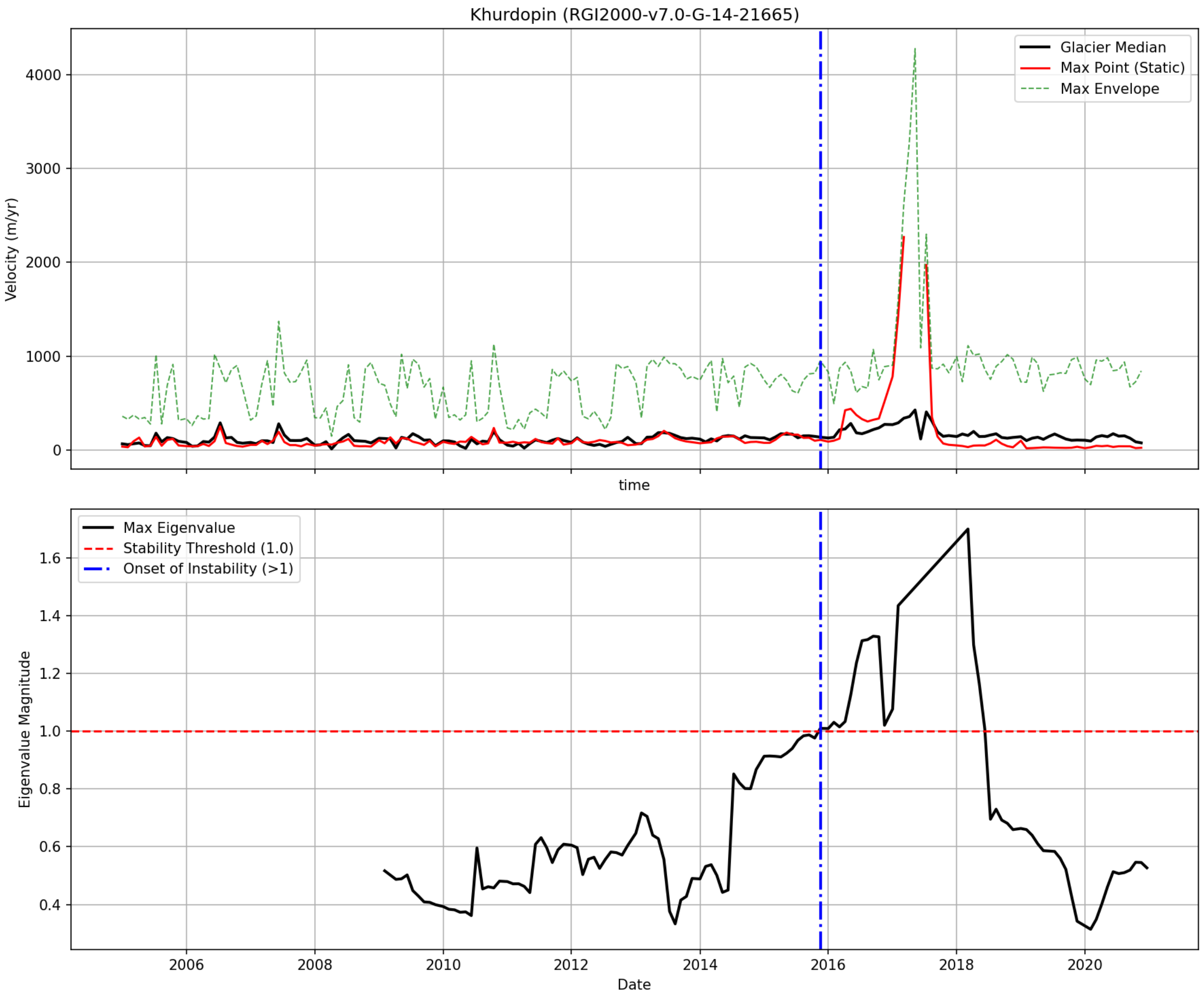
Early Warning via DMD / Koopman Theory

- Use Dynamic Mode Decomposition (DMD) on a sliding window over the time series
- Each window provides a local linear approximation of system dynamics
- Compute Dynamic Eigenvalue (DEV):
  - Eigenvalue of the DMD-estimated local linear operator
  - Approximates eigenvalues of the underlying system Jacobian
- Interpretation:
  - |DEV| approaching 1 from below -> loss of stability
  - Crossing 1 -> onset of instability
- Tracking DEV over time reveals evolving system stability

Smith, T. *et al.* (2026) *Predicting instabilities in transient landforms and interconnected ecosystems*, *Nature Communications*.

Case Study: Khurdopin Glacier

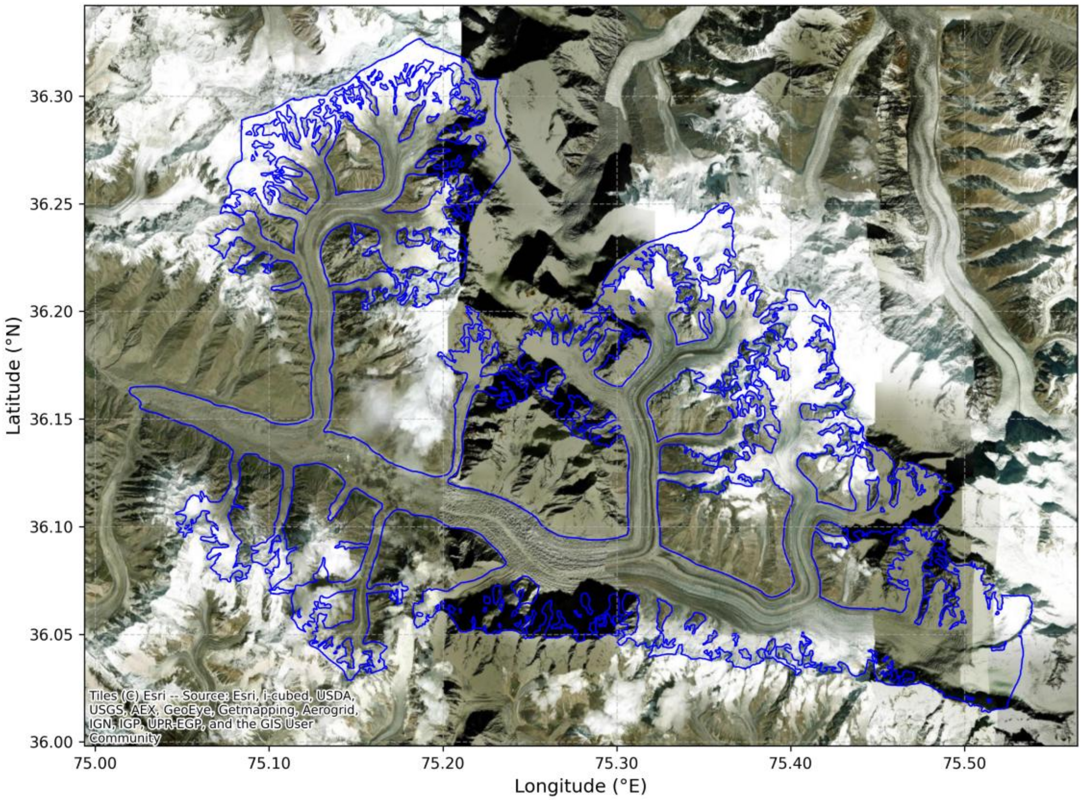


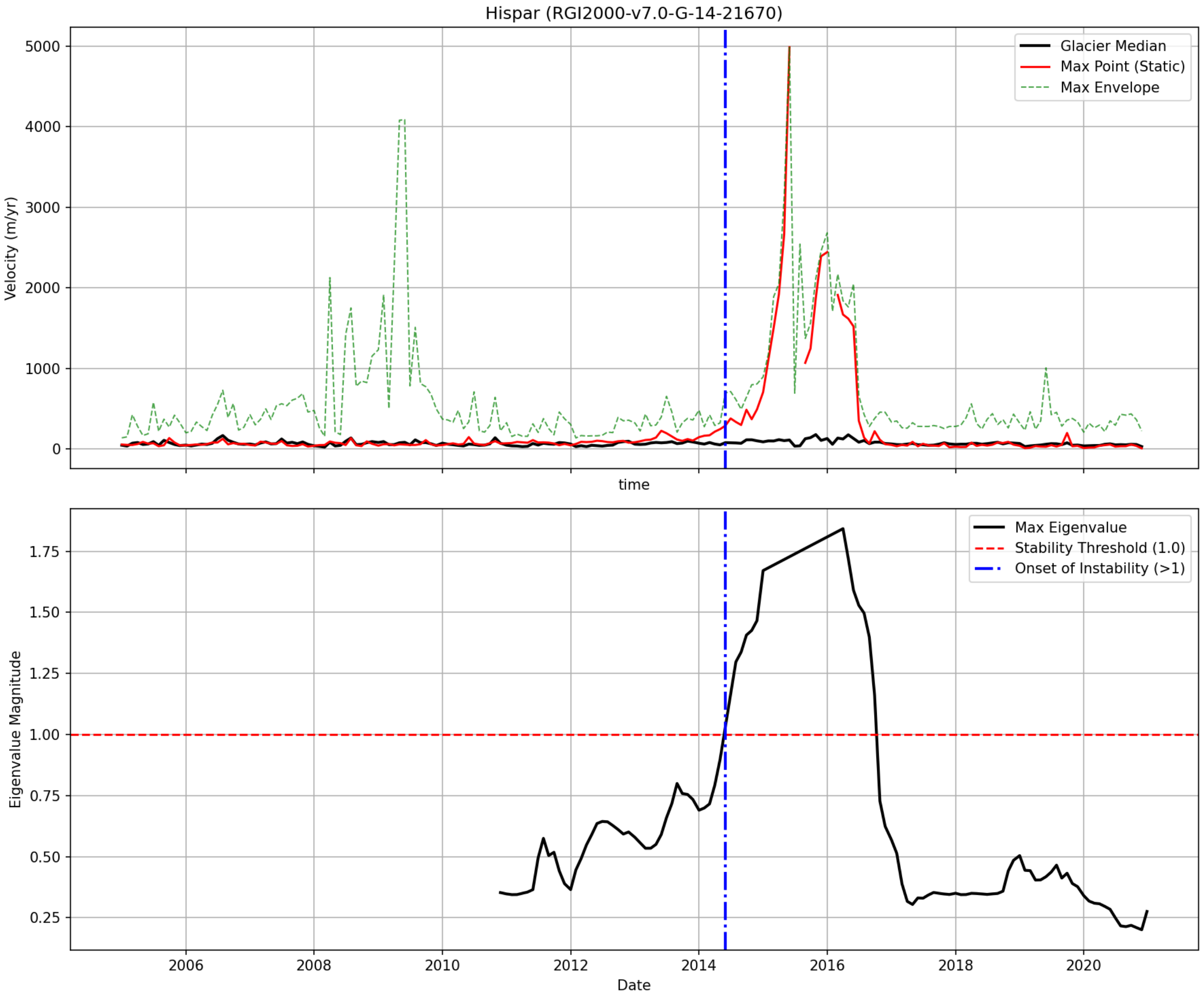


- Large surge-type glacier (approx. 40 km length) in Shimshal Valley, Karakoram, Pakistan
- Surged in 2017 blocking the Shimshal river
- Upcoming surge predicted a year in advance

Steiner, J.F. *et al.* (2018) *The Khurdopin Glacier Surge Revisited – extreme flow velocities and formation of a dammed lake in 2017, The Cryosphere*.

Case Study: Hispar Glacier





- Large surge-type glacier (approx. 50 km length) in Karakoram, Pakistan
- Began surging in October 2014, peaking in 2015
- Surge predicted a few months in advance

Paul, F. *et al.* (2017) *The 2015 surge of Hispar Glacier in the Karakoram, MDPI.*

Earth Systems Dynamics with a Social Network

- The Sociological Dynamics represents an individual community that is influenced by other communities in its proximity.
- The Ecological Dynamics represents a common pool resource on which the communities rely.
- Under certain structural assumptions, the Individual and Community Level interaction structures are similar.
- Unknown parameters: $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{p}, \boldsymbol{v}$ and $\boldsymbol{w}$, represent the interaction structures within the communities i.e. sociological and ecological influences and preferences.
- Why are these parameters important?
  - Policy and Analysis!
- How to infer these parameters?
  - Data-Driven Method: Dynamic Mode Decomposition with Control

Sociological Dynamics

$$\dot{y}_i(t) = n_i b_i \left(a_i(x(t) - p_i) - v_i \sum_{j=1}^{n} w_{ij}(y_i(t) - y_j(t)) \right)$$

Ecological Dynamics

$$\dot{x}(t) = x(t)\big(1 - x(t)\big) - x(t) \sum_{i=1}^{n} y_i(t)$$

Constraints

$$\sum_{j=1}^{n} w_{ij} = 1, \quad w_{ii} = 0, \quad w_{ij} \geq 0, \quad \forall i, j$$

* Manzoor, T., Rovenskaya, E., and Muhammad, A. (2017). Structural effects and aggregation in a social network model of natural resource consumption. IFACPapersOnLine, 50(1), 7675–7680.

Matrix Representation

- The Sociological Dynamics are affine; hence we can obtain its matrix representation suitable for DMDc.
- Let: $b'_i = n_i b_i$

$$\dot{\mathbf{Y}}(t) = \underbrace{\begin{bmatrix} b'_1 a_1 \\ b'_2 a_2 \\ \vdots \; b'_n a_n \end{bmatrix}}_{\mathbf{B'A1}} \mathbf{x}(t) - \underbrace{\begin{bmatrix} b'_1 a_1 p_1 \\ b'_2 a_2 p_2 \\ \vdots \; b'_n a_n p_n \end{bmatrix}}_{\mathbf{B'AP1}} - \underbrace{\begin{bmatrix} b'_1 v_1 & 0 & \cdots & 0 \\ 0 & b'_2 v_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & b'_n v_n \end{bmatrix}}_{\mathbf{B'V}} \underbrace{\begin{bmatrix} 1 & -w_{12} & \cdots & -w_{1n} \\ -w_{21} & 1 & \cdots & -w_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ -w_{n1} & -w_{n2} & \cdots & 1 \end{bmatrix}}_{\mathbf{T}} \underbrace{\begin{bmatrix} y_1 \\ y_2 \\ \vdots \; y_n \end{bmatrix}}_{\mathbf{Y}}$$

$$\dot{\mathbf{Y}}(t) = -\mathbf{B'VTY}(t) + \mathbf{B'A1_m x}(t) - \mathbf{B'AP1_m}$$

$$\dot{\mathbf{Y}}(t) = \underbrace{-\mathbf{B'VT}}_{\mathbf{F}}\mathbf{Y}(t) + \underbrace{\begin{bmatrix}\mathbf{B'A1_m} & -\mathbf{B'AP1_m}\end{bmatrix}}_{\mathbf{G}} \begin{bmatrix} \mathbf{x}(t) \\ 1 \end{bmatrix}$$

Dynamic Mode Decomposition with Control (DMDc)*

- DMDc leverages snapshots of the system's states to learn a best fit linear operator
  - Input/Output model incorporates feedback (this preserves the socio – ecological coupling)
  - $\boldsymbol{Y_{k+1} = F_d Y_k + G_d x_k}$
- Purpose: System Identification
  - Learn DT matrices $\boldsymbol{F_d}$ and $\boldsymbol{G_d}$
- Original System is CT, a conversion to DT (via ZOH) is required
- Advantages
  - Reduced Order System: Filter out spurious correlations and Identify dominant users.
  - Extraction of Spatio-Temporal Modes: Insights into the system's behavior

---

Algorithm 1 Dynamic Mode Decomposition with Control (DMDc) (Proctor et al., 2016)

Require: Collect and construct snapshot matrices. Let s be the number of snapshots

$$\mathbf{Y} = [y_1, \ldots, y_{s-1}]$$
$$\mathbf{Y}^+ = [y_2, \ldots, y_s]$$
$$\mathbf{x} = [x_1, \ldots, x_{s-1}]$$

1: Augmented data matrix

$$\boldsymbol{\Omega} = \begin{bmatrix} \mathbf{Y} \\ \mathbf{x} \\ 1 \end{bmatrix}$$

2: SVD of Input Space $\boldsymbol{\Omega}$: truncation value p

$$\boldsymbol{\Omega} \approx \tilde{\mathbf{U}}\,\tilde{\boldsymbol{\Sigma}}\,\tilde{\mathbf{V}}^*$$

3: SVD of Output Space $\mathbf{Y}^+$: truncation value r

$$\mathbf{Y}^+ \approx \hat{\mathbf{U}}\,\hat{\boldsymbol{\Sigma}}\,\hat{\mathbf{V}}^*$$

4: Approximation of the Operators

$$\bar{\mathbf{F}} = \mathbf{Y}^+\tilde{\mathbf{V}}\tilde{\boldsymbol{\Sigma}}^{-1}\tilde{\mathbf{U}}_1^*$$
$$\bar{\mathbf{G}} = \mathbf{Y}^+\tilde{\mathbf{V}}\tilde{\boldsymbol{\Sigma}}^{-1}\tilde{\mathbf{U}}_2^*$$
$$\tilde{\mathbf{F}} = \hat{\mathbf{U}}^*\bar{\mathbf{F}}\hat{\mathbf{U}}$$
$$\tilde{\mathbf{G}} = \hat{\mathbf{U}}^*\bar{\mathbf{G}}$$

5: Eigen-decomposition

$$\tilde{\mathbf{F}}\mathbf{W} = \mathbf{W}\boldsymbol{\Lambda}$$

6: Compute DMD modes

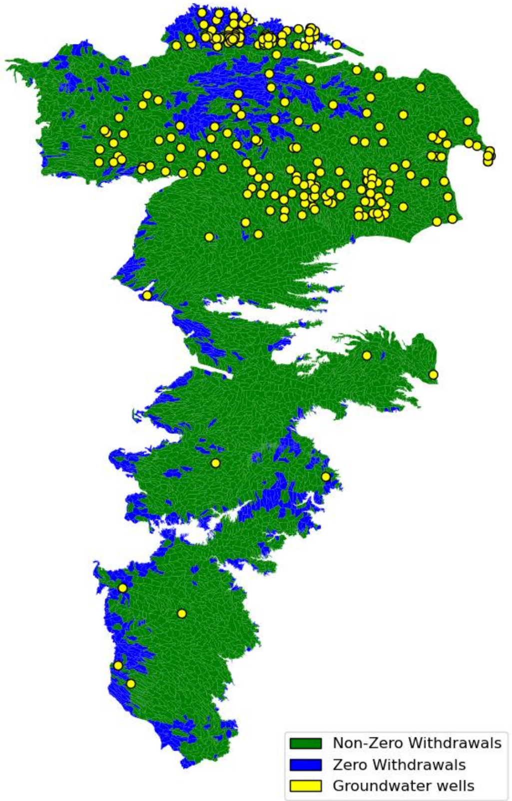
$$\boldsymbol{\Phi} = \mathbf{Y}^+\tilde{\mathbf{V}}\tilde{\boldsymbol{\Sigma}}^{-1}\tilde{\mathbf{U}}_1^*\hat{\mathbf{U}}\mathbf{W}$$

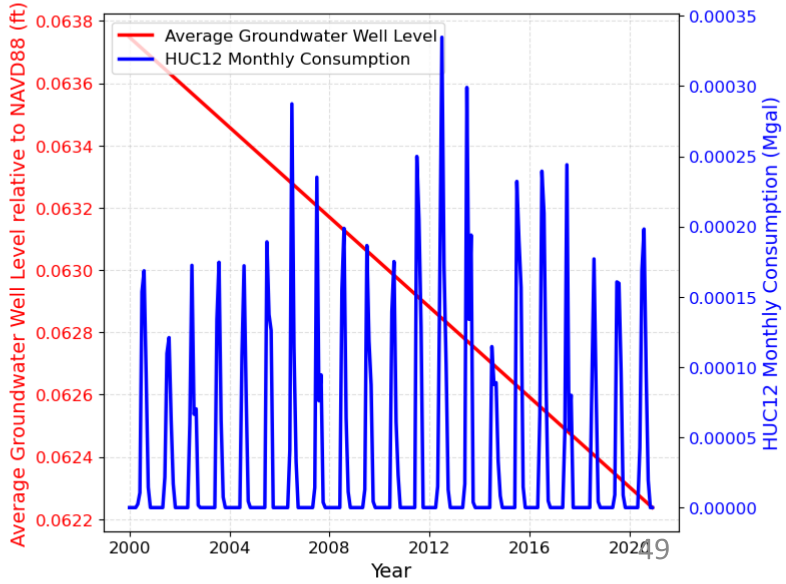
---

* Proctor, J.L., Brunton, S.L., and Kutz, J.N. (2016). Dynamic mode decomposition with control. SIAM Journal on Applied Dynamical Systems, 15(1), 142–161. doi: 10.1137/15M1013857.

Groundwater Data

- High Plains Aquifer (174,000 sq mi)
  - 5002 Sub-watershed Regions (HUC12): Hydrological and not Sociological boundaries (3868 have non-zero consumption)
- Consumption Data**
  - Monthly estimates of Public Supply + Irrigation withdrawals from Jan 2000 till Dec 2020 in Mgal per HUC12
- Groundwater Level Data* (proxy for stock)
  - Inconsistent readings of water depth (ft) relative to NAVD88
  - Average Representation is assumed to be valid for consumers in proximity
- Determining a system with 3868 states with 252 data points!!

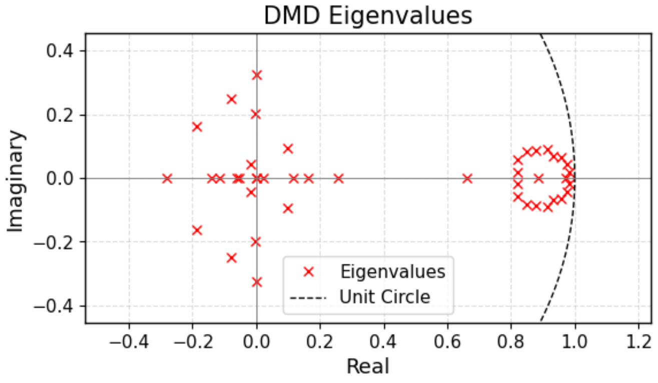


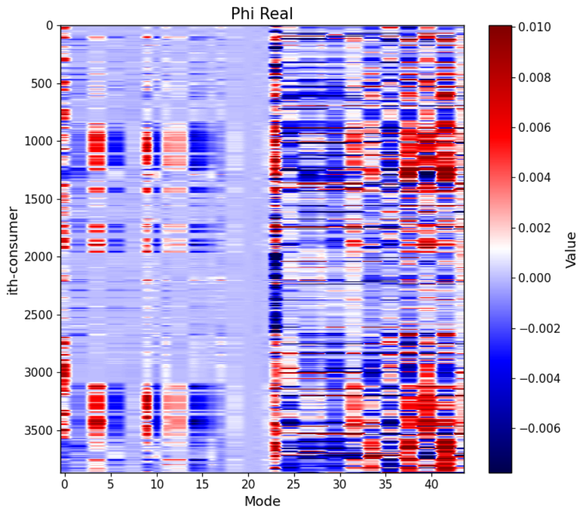


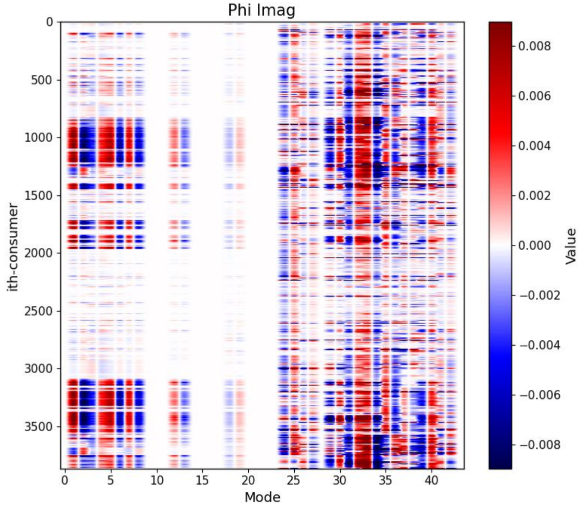
* U.S. Geological Survey (2025). National Ground-Water Monitoring Network Data Portal. Accessed: 2025-08-15.
** U.S. Geological Survey, Water Resources Mission Area (2023). Water Use in the United States. Accessed: 2025- 08-15.

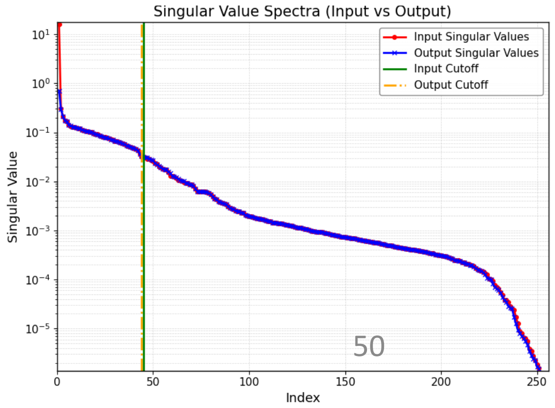
The Results

- Reduced Order Model Rank
  - **99%** Energy Cutoff of Singular Values
    - 42 from possible 3868!
  - Output space rank is lower than input space rank
- DMDc Operator’s ($\boldsymbol{F}_d$) Eigenvalues are inside the unit circle
- Complex DMDc Modes: Interpretation
  - Slow vs Fast changing modes. Slow means dominant, but fast modes encompass a broader range of consumers.
    - Public consumption has a different pattern and is less significant than Irrigation consumption.
  - A few consumers derive significant change in the groundwater dynamics i.e. irrigators. Irrigated regions have close proximity – hence clustering in modes.
  - Parameter Identification: Requires the full order system lifted from the reduced order system.

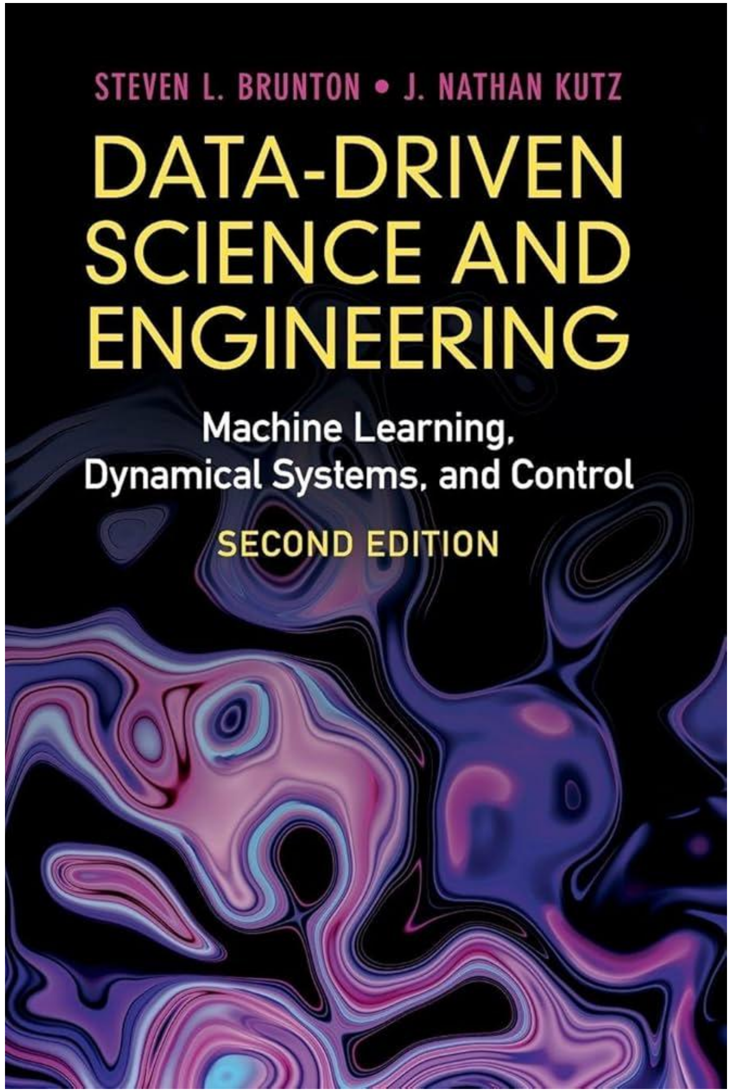




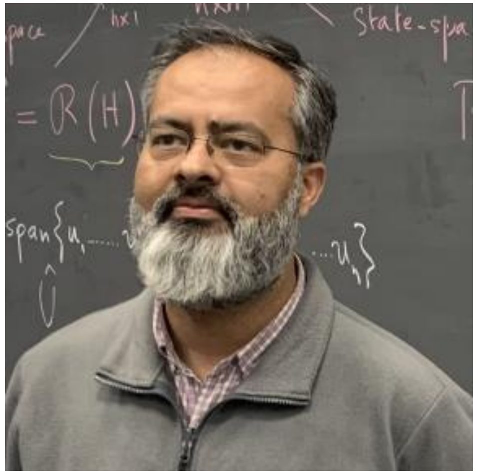




To Learn More ....



Annual Conference (L4DC): https://sites.google.com/usc.edu/l4dc2026/



EE5614. Learning for Dynamics & Control, LUMS. Summer 2026.

Thank you!

LUMS Centre for Water Informatics and Technology

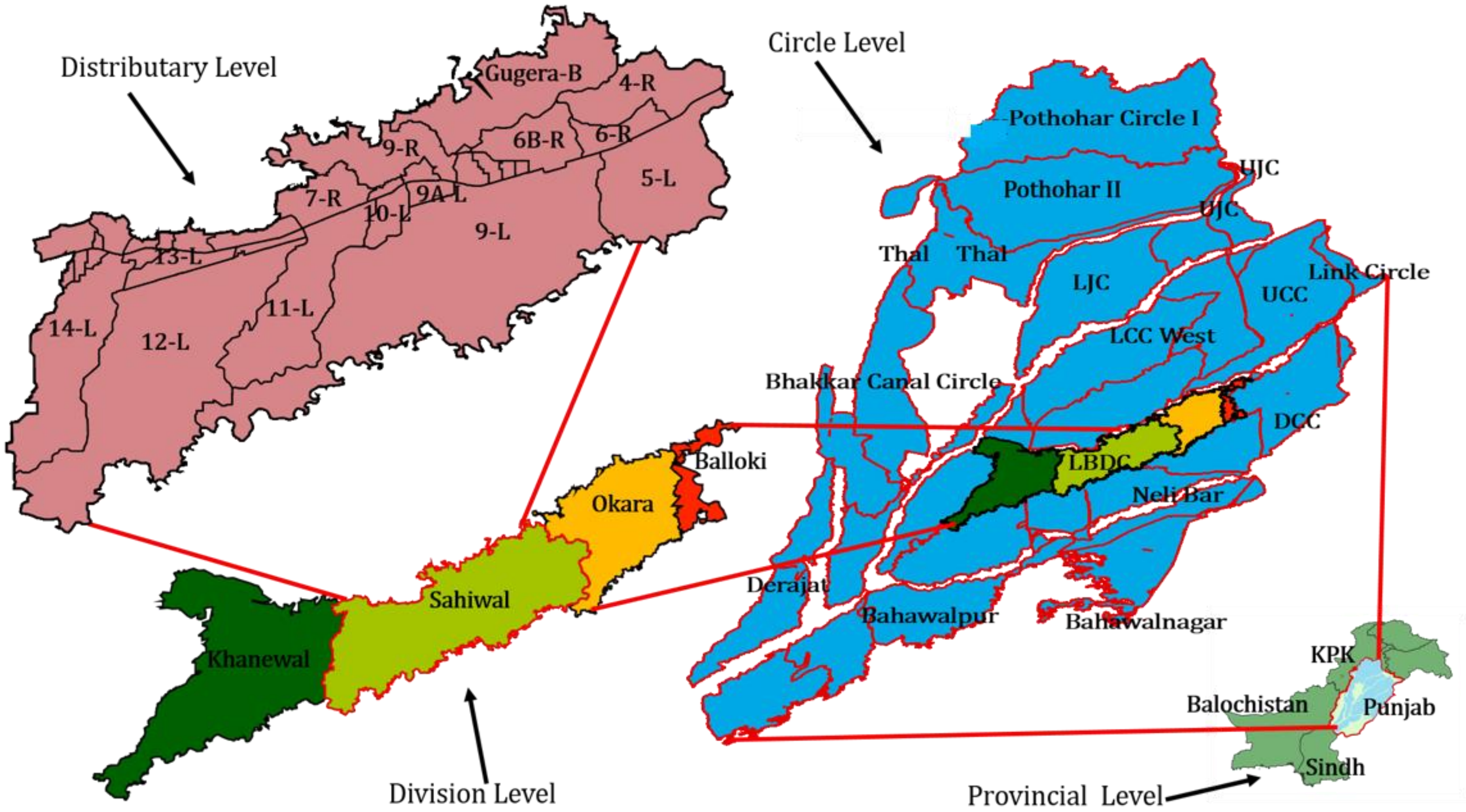
Improving Canal Irrigation Management through Remote Sensing Based Decision Support Tool

Demand management by technology driven Water accounting Precision irrigation advisory services

Poor water management costs 4 percent of GDP or around $12 billion per year. (WB)

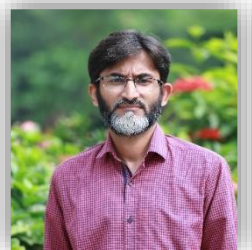
1 MAF = economic worth $1 billion

LBDC (10,000 Cusecs) ~ 7 MAF
Punjab SW ~ 56 MAF





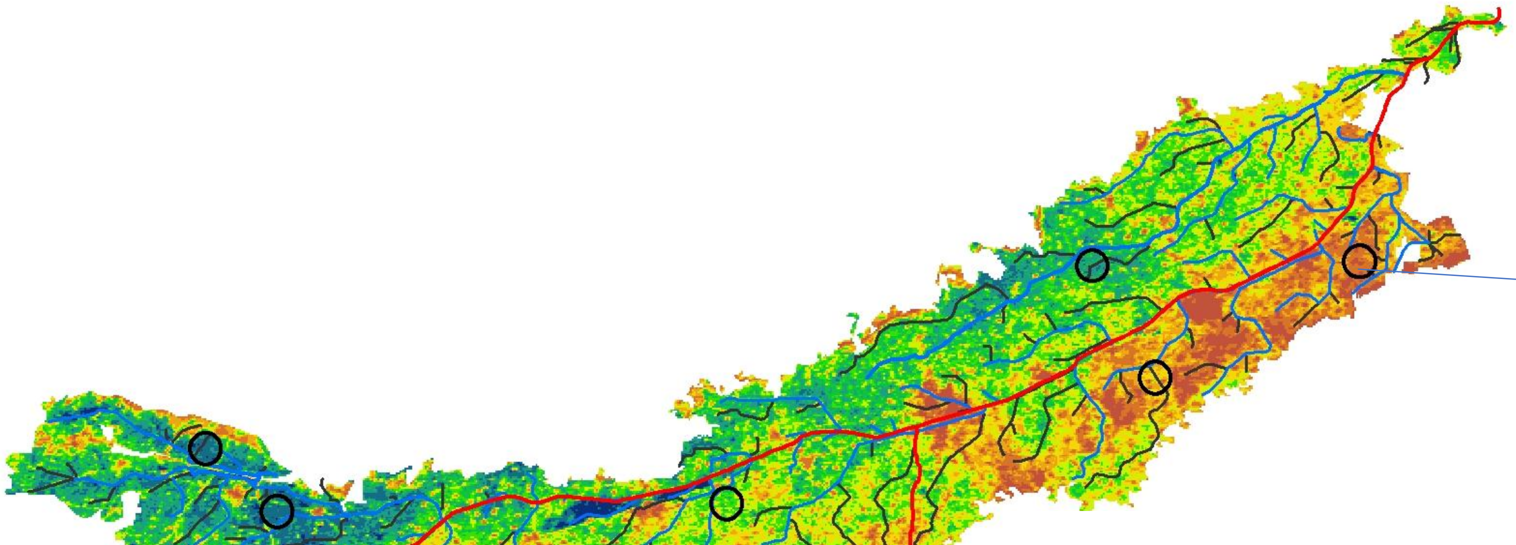
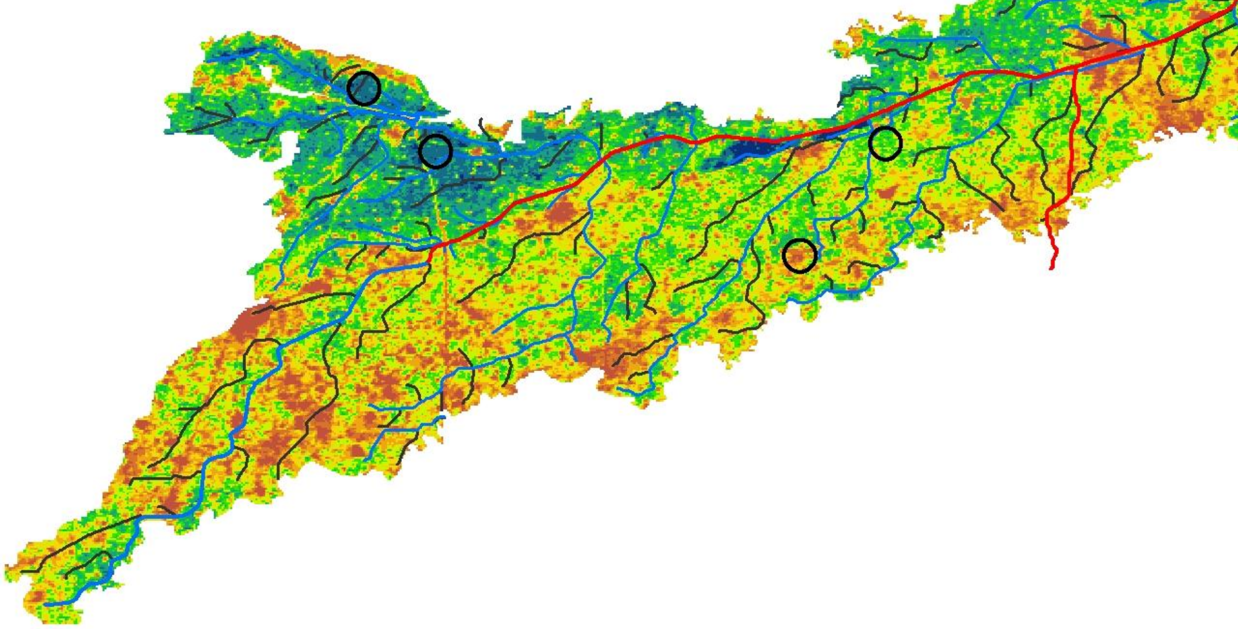


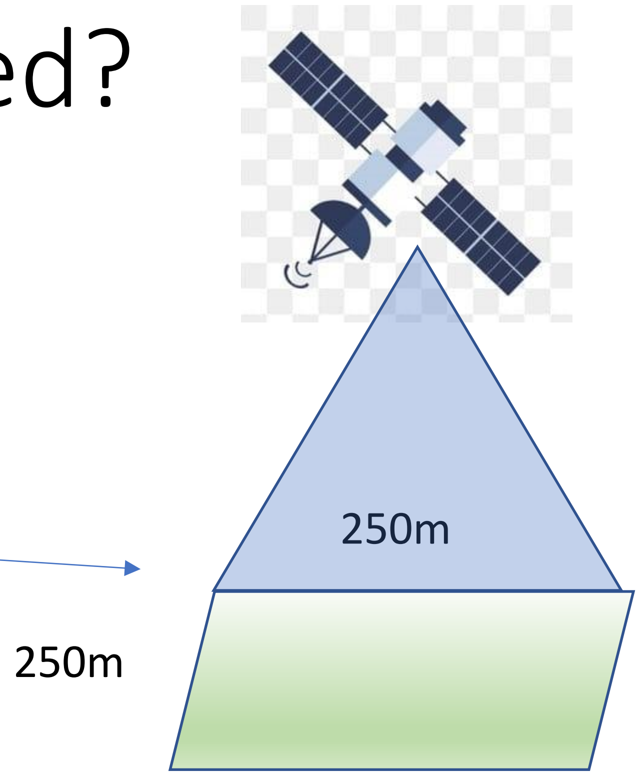



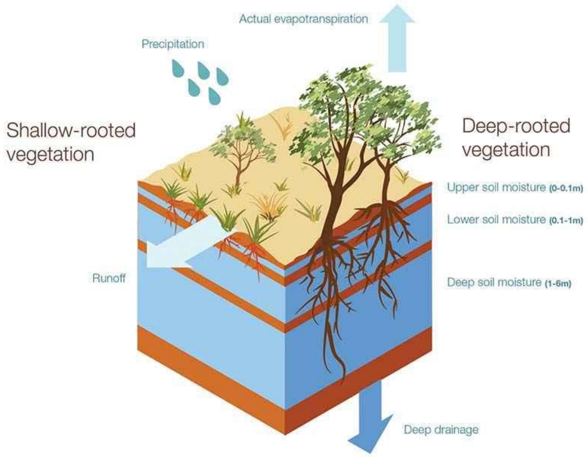
IRRIGATION Department Punjab

ADB

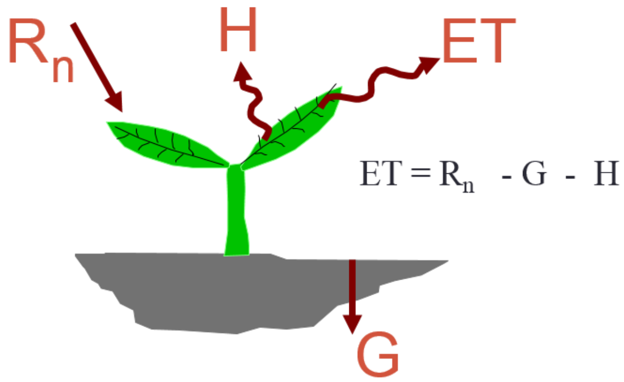
What much does the crop really need?

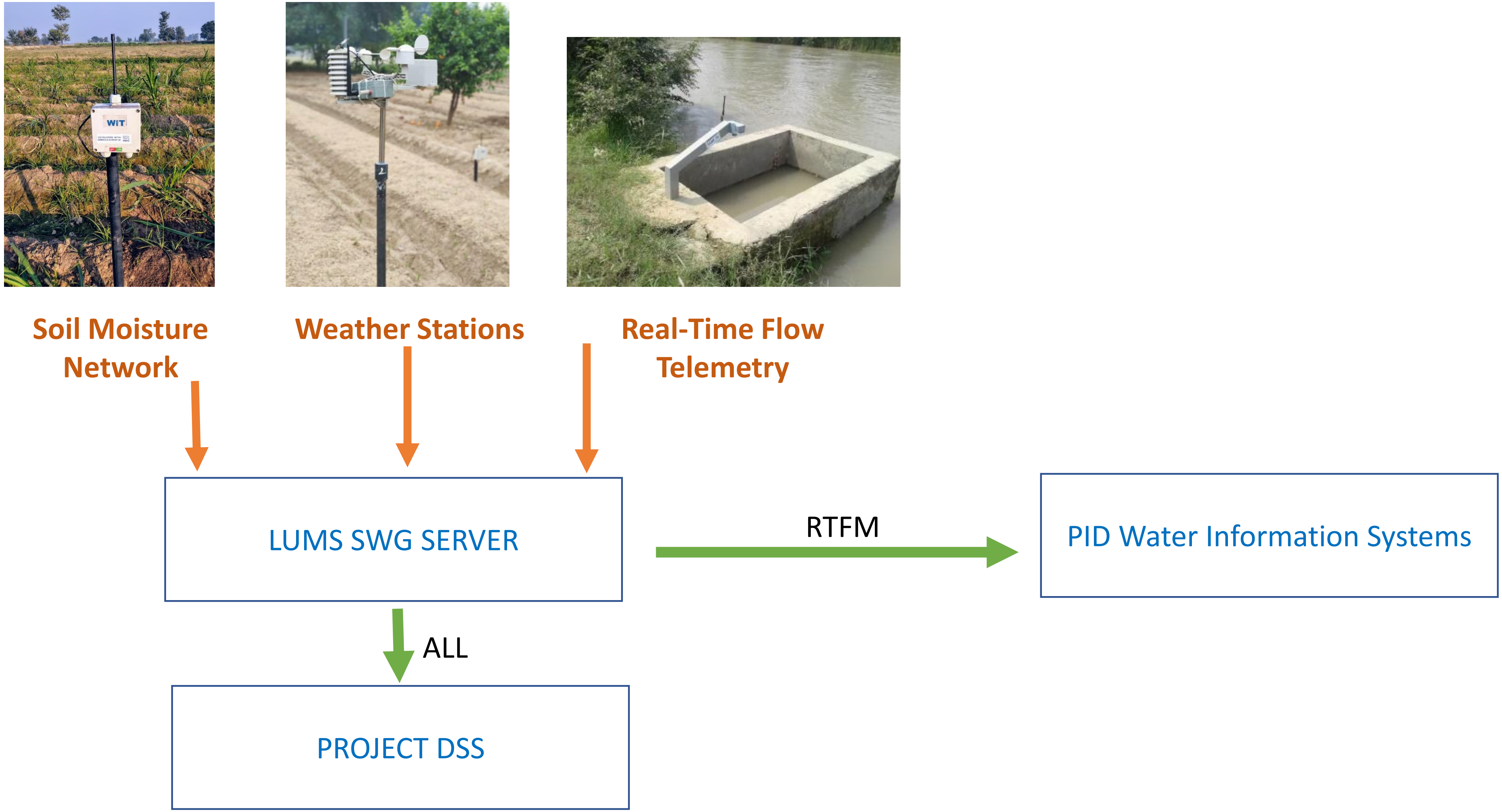




Water Balance



Energy Balance


WiT
Soil Moisture Network
Weather Stations
Real-Time Flow Telemetry
LUMS SWG SERVER
RTFM
PID Water Information Systems
ALL
PROJECT DSS

IoT based Interventions

Cryosphere Research for Basin Scale Water Availability Assessment

Digital Agriculture with Soil Moisture Sensors

Integrated Irrigation Water Management

Building Resilience in Data Scarce Watersheds


5.11
Nikon

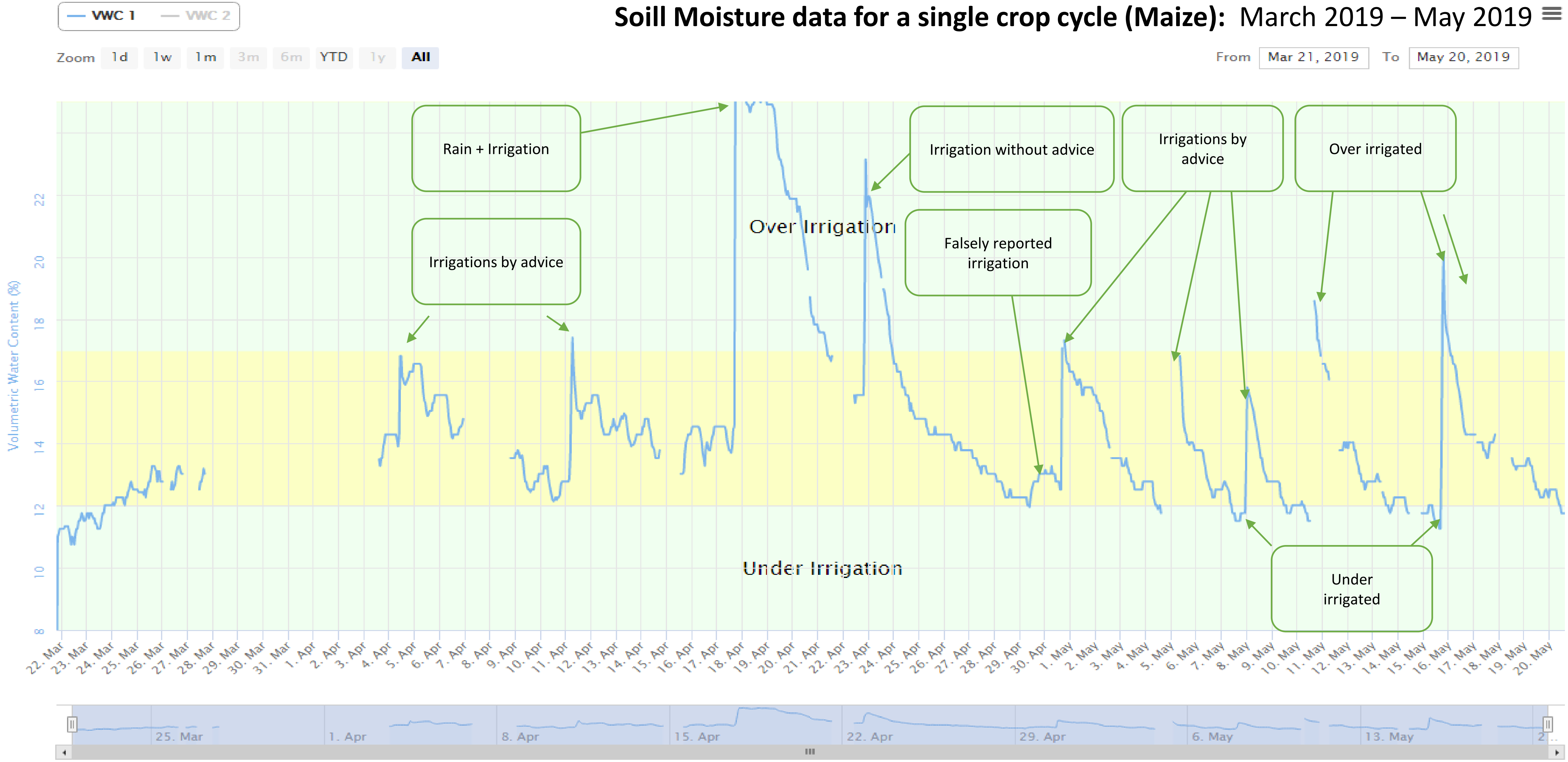
Hundreds of active Installation Sites at Small & Medium Sized Farms

xWIT

SMART WATER SOLUTIONS

Farmer Behavior & Technology Adoption

Soill Moisture data for a single crop cycle (Maize): March 2019 – May 2019



Smart Water Grids (SWG) developed by Center for Water Informatics & Technology (WIT), LUMS